A Universe of Proportions:
Scaling and Ratios in Physics and Beyond

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Outline

- Introduction
- Analogies
- Ratios and Proportions
- Projections
- Relations and Equations
- Examples and Insights
- Conclusion

Introduction

- Observations
- Ratios and Comparisons of Change
- Analogies and Inference
- Cognition and Compartmentalization
- Natural Philosophy
- Physics

Analogies

- Comparison between two forms that may share a property
  - At the core of cognition
- Logical inference rather than abduction, induction, or deduction
- The word “analogy” comes from the Latin “analogia” and originates from the Greek word for proportion, ἀναλογία
- Logic: A is to B as C is to D
  - A:B = C:D or A:B::C:D

pj evans

Cars have windows and can move. Houses have windows and can't move. So It's not the windows that make the car go, It's something else entirely

gelledegg

this is what ancient greek philosophy is like

Ratio and Proportions

- A ratio is the comparison of, or relationship between, two values
- A ratio is a proportion, and a proportionality is the equality of ratios
- Rational – Ratio (Reason) [or computation/reckoning]
- The ratio A:B is proportional to the ratio C:D

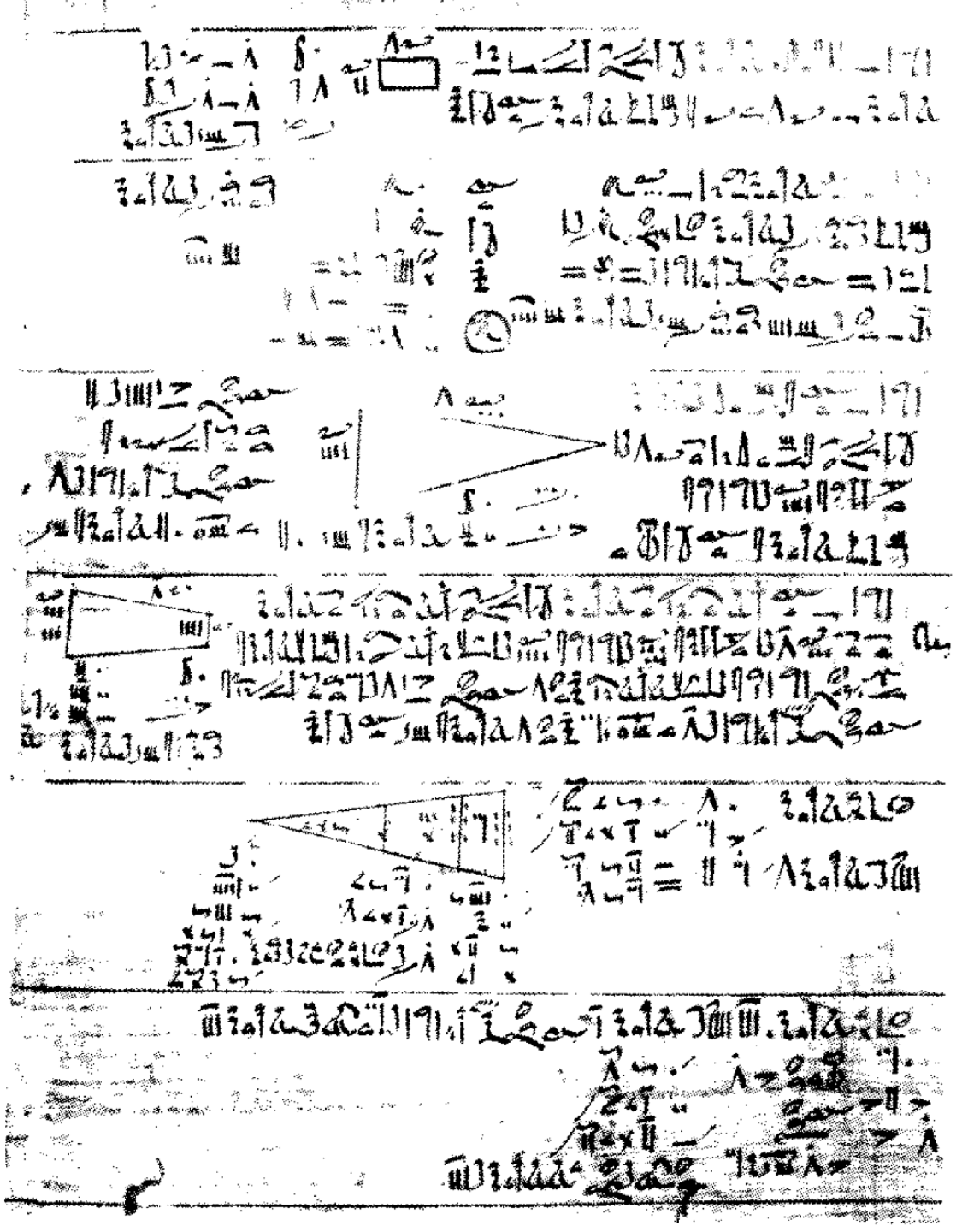
$$\frac{A}{B} = \frac{C}{D}$$

Geometry

- Consider two similar right triangles
- The lengths of their sides are different, and angles are same
- The ratio of the lengths of their respective sides is proportional
- Some aspect of the angle is proportional to these ratios
- Sine, Cosine, Tangent
- Sine is the opposite or perpendicular proportion (projection)
- Cosine is the adjacent or parallel proportion (projection)
- Tangent is the ratio of these proportions!

Projections

- Ratios of lengths of triangle sides
  - Plimpton 322 – Babylon ~1800 BC
- Seked – Slope
  - Rhind Papyrus - Egypt ~1550 BC
  - Estimated π as 256/81
- Sun projects a shadow of a figure
  - Sundial ~1500 BC
- Theory of Proportions
  - Pythagoras ~500 BC (RIP Hippasus of Metapontum)
- Sine, Cosine, and Tangent (Tables)
  - Al-Khwarizmi ~9th century



Relations and Equations

- Proportions -> Relations -> Equations
- But wait, not everything scales linearly!
- Consider a bunch of little triangles/slopes.. (differentials!)
- Some values scale as a ratio of ratios
- The tangent function is a ratio of ratios!
- Equations are essentially complex rational proportionalities
  - (and are approximations!)

Examples of Ratios and Proportions

- Pi (ratio of the circumference of a circle to its diameter)
  - $\pi = \frac{C}{d} = \frac{2\pi r}{2r}$
- Original units of time and length
  - 1/86,400 of a mean solar day, 1/10,000,000 distance from equator to NP
- Pythagorean Tuning (Perfect Fifths)
  - 3:2
- Archimedes Principle ($F_B = W_{fluid}$)
  - Fraction Submerged: $\frac{V_{sub}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}} = \frac{F_W}{F_B}$
  - Unknown object density experiment: $\rho_o = \frac{T_a}{T_a - T_b}\rho_w$
- Center of Mass/Gravity (Homogenous Materials)
  - $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$
- Mechanical Advantage (Simple Machines)
  - $MA = \frac{F_O}{F_i} = \frac{r_i}{r_o}$

Examples of Ratios and Proportions

- Kepler's Third Law (Law of Harmonies)
  - $\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$
- Pascal's Principle
  - $\frac{F_1}{F_2} = \frac{A_1}{A_2}$
- Newton's Second Law
  - $F = \frac{\Delta p}{\Delta t}$
- Kinetic Energy and Velocity
  - $\frac{KE_i}{KE_f} = \frac{v_i^2}{v_f^2}\ (for\ same\ mass)$
- Energy and Efficiency
  - $\varepsilon = \frac{\Delta KE}{\Delta PE} = \frac{W_{out}}{E_{in}}$
- Angular Speed and Torque (Gear Ratio)
  - $\frac{\omega_1}{\omega_2} = \frac{\mathrm{T}_2}{\mathrm{T}_1} = \frac{d_2}{d_1}$

Examples of Ratios and Proportions

- Gas Laws
- Relative Humidity
  - $\% \, RH = \frac{vapor \; density}{saturation \; vapor \; density} \times 100\%$
- Specific Heat
  - $c_B = \frac{\Delta T_A}{\Delta T_B} c_A$
- Decibel Scale
  - $\mathrm{B} = 10 \log_{10} \frac{I}{I_o}$
- The Doppler Effect
  - $f_{obs} = \frac{v_w}{v_w \pm v_s} f_s$

Examples of Ratios and Proportions

- Inclined Plane Problems (Ratio of weight parallel and perpendicular to plane)
- Simple Collision Problems (Conservation of Momentum)
  - $\frac{m_1}{m_2} = \frac{v_2}{v_1}$
- Simple Tension Equilibrium Problems
  - $\frac{T_1}{T_2} = \frac{\cos\theta_2}{\cos\theta_1}$
- Gravitational Force and Electric Force Equilibrium Problems
  - $\tan\theta = \frac{F_E}{F_G} = \frac{qE}{mg}$
- Electrical Resistance (Ohmic), Conductivity, and Resistivity
  - $R = \frac{V}{I}, \sigma = \frac{\vec{J}}{\vec{E}}, \rho = \frac{1}{\sigma}$
  - $\frac{R_2}{R_1} = \frac{L_2 r_1^2}{L_1 r_2^2}$ (resistance change via expanded wire length, constant volume)
- Charged Particles Moving Through Magnetic Field
  - $\frac{F_1}{F_2} = \frac{v_1 \sin\theta_1}{v_2 \sin\theta_2}$
- Fick's Law (Diffusion flux is proportional to the diffusion coefficient and the concentration gradient)
  - $J = -D\frac{d\varphi}{dx}$

Ratios in Physics (Constants & Coefficients)

- Speed of Light
  - $c = \frac{E}{B}$
- Friction Problems (Coefficient of Friction)
  - $\mu_k = \tan(\theta) = \frac{v^2}{rg}$
- Collision/Fluid Flow Problems (Coefficient of Restitution/)
  - $e =$
- Optics (Index of Refraction)
  - $n_m = \frac{c}{v_m} = \frac{\lambda_o}{\lambda_m} = \frac{\sin\theta_o}{\sin\theta_m} n_o$
- Springs (Stiffness Coefficient)
  - $k =$
- Dampeners (Dampening Coefficient)
  - $\zeta =$
- Dielectric Constant
  - $\kappa = \frac{E_o}{E}$

Ratios in Physics

- Electric Field Strength Proportional to Charge or Field Line Density
- System Stability
- Convergence and Divergence

Ratios Beyond Physics

- Statistics:
  - % Error and % Difference
- Mathematics:
  - Golden Ratio
  - Fibonacci Sequence
  - Distance of Planets
    - https://www.fq.math.ca/Scanned/8-4/read-a.pdf

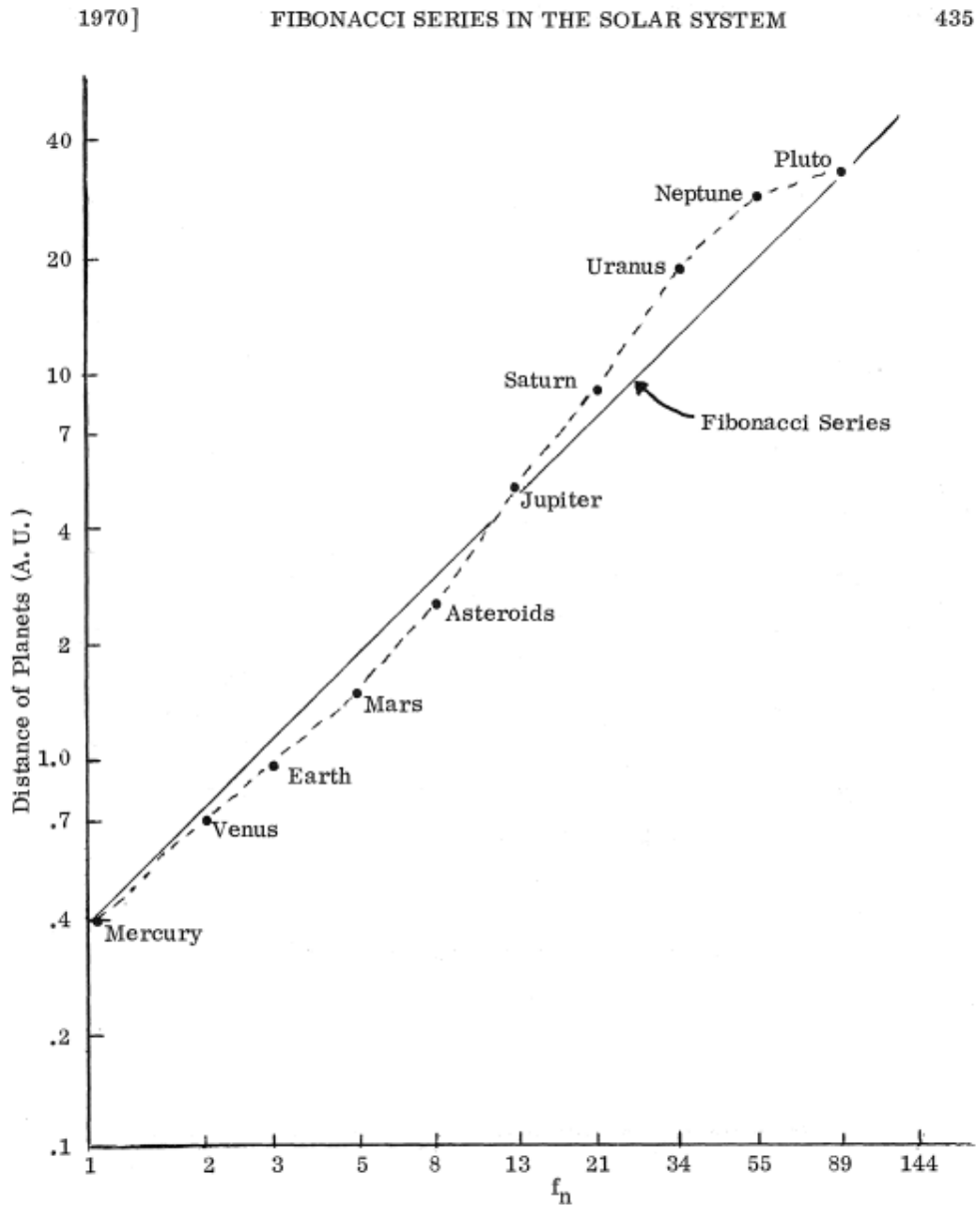
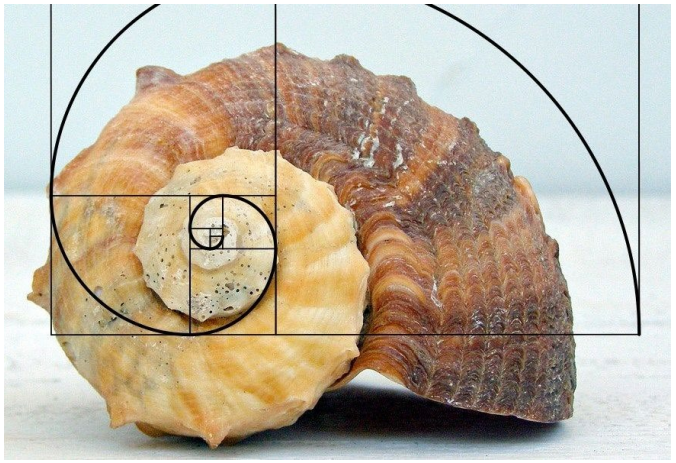
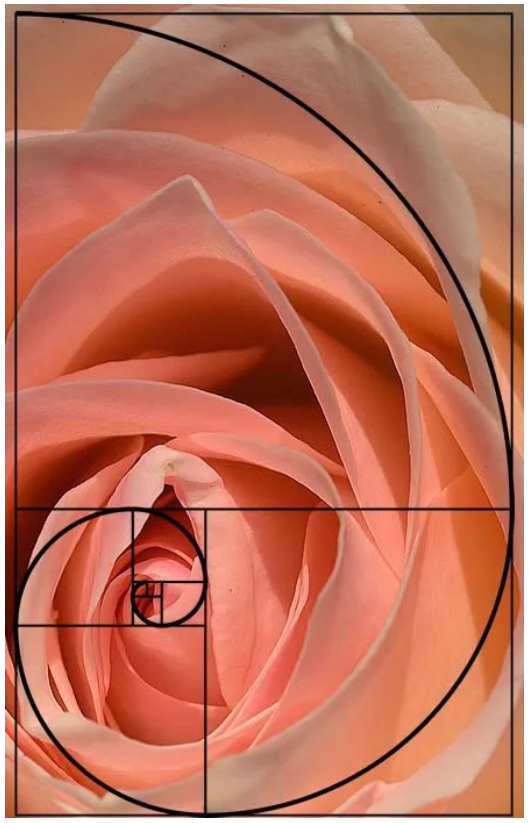
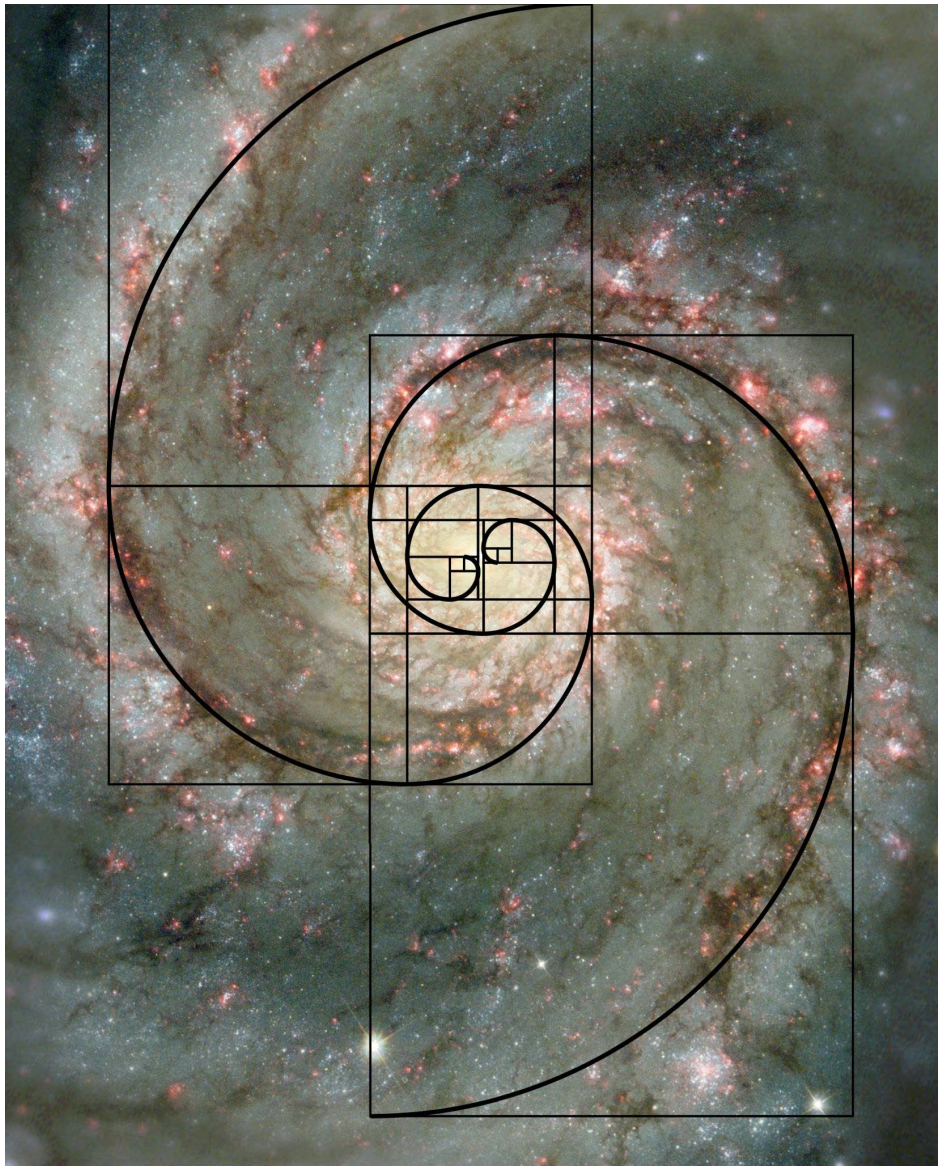
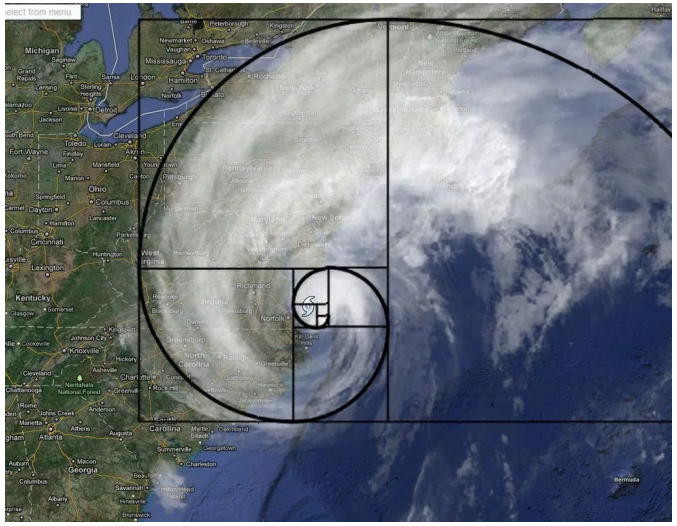


Fig. 6 The Planets








Michigan
Toronto
Ohio
Kentucky
Georgia
Bermuda

Ratios Beyond Physics

- Biology:
  - Genetic Ratios (Mendel's Law of Segregation)
  - Convergent Evolution
  - Surface Area to Volume Ratio
  - Population Growth
  - Photosynthesis and Light Intensity
- Chemistry:
  - Gas Laws
  - Stoichiometry (ratios of reactants and products)
  - Reaction rate and concentration (rate proportional to concentration)
  - Molar mass ratios

Summary

- Anything can be expressed as a ratio so long as the compared objects/parameters are dimensionally equivalent
- Ratios are easy to create, proportionalities are difficult to establish

Questions?