Cole Prather 19 June 2024

On Physical Analogies

Warning to reader: Work in progress. May be redundant and scattered.

I. Introduction

During my time as a lab instructor, I tried to reconcile the terminology of "Electromotive Force" used often in textbooks and in our lab manual. You may already be aware that the EMF is not even a force! EMF has units of Joules/Coulomb - the same as electric potential, with units of energy per unit of charge. Indeed, electric potential energy can be found by multiplying the potential by the charge and the result will have units of energy (Joules). However, the name "Electromotive Force" implies that this physical parameter would have units of force – Newtons. Rather, it represents a potential. While working on my thesis, I attempted to rationalize the units of electric potential and gravitational potential to realize a comprehensible analogy of the latter to the former. Although the result may be rather obvious in hindsight (which is 20/20), I believe it worthwhile to document my findings and how I came to my results thus far, for there is much more to discern.

In summary, I am attempting to deduce a viable analogy between gravitation and electromagnetism that considers a set of basic parameters including displacement, velocity, acceleration, sources, fields, forces, energies, and potentials. This analogy is based on the writings of Oliver Heaviside and his work on describing gravitational waves and how they should relate directly to electromagnetic waves. In fact, Heaviside proposes that there exists a component field of the static gravitational field, or *gravitic* field, that is analogous to the magnetic field. This secondary gravitational field is called the cogravtiational field. For Heaviside's namesake, this field will be referred to as the *heavitic field*, the gravitational analog to the magnetic field. To describe gravitational effects more fully, the term *gravitoheavitic* will be incorporated in the lexicon so to reflect the interaction of both fields and to reflect Heaviside's contribution to the study of gravity and gravitational waves.

I.1 Sources, Fields, and Forces

The source units of gravitation and electromagnetism are mass - with units of kilograms [kg] - and charge - with units of Coulombs [C]. Gravitational mass generates a radially inward gravitational field, or gravitic field, and the motion of mass generates a rotationally positive heavitic field. Electric charge generates a radially outward electric field, and the motion of a charge generates a rotationally positive magnetic field.

Gravitational masses interact with gravitoheavitic fields and electrical charges interact with electromagnetic fields. In electromagnetism, there are the electric and magnetic fields. In gravitation, there are the gravitic and heavitic fields. These fields are acceleration fields, with the gravitic or gravitational field, g, measured in meters per second squared [m/s^2] or Newtons per kilogram [N/kg] and the electric field, E, measured in Volts per meter [V/m] or Newtons per Coulomb [N/C]. Notice that the field dimensions are similarly related by force units per source units.

Gravitational masses in a gravitational field experience a force given by:

$$F = mg,$$

where F is the force, m is the mass, and g is the acceleration due to the gravitational field. More generally, a mass experiencing any acceleration experiences a proportional force, expressed as:

$$F = ma,$$

where F is the force, m is the mass, and a is the acceleration.

Electrical charges in an electrical field experience a force given by:

$$F = qE,$$

where F is the force, m is the mass, and E is the acceleration due to the electric field.

Notice that the force is similarly determined by multiplying the source with the field. Force, in both cases, is measured in Newtons [N]. In general, Sources interact with Fields and experience Forces.

I.4 Energies

Moving a source through a field with force requires energy. A relatable form of energy is work, which is simply the ability of a force to move things about. The work done by a force to move a source over some distance is given by:

$$W = Fdcos\theta,$$

where W is measured in Joules [J], F is the force measured in Newtons [N], d is the distance in meters [m], and θ is the angle between the applied force and the line of action (direction of motion).

Note that (optimal force vs suboptimal force) and assume optimal force application

Similarly, the concept of potential energy is a form of energy and potential energy is also calculated using force and distance. Gravitational potential energy can be determined using the equation:

$$U = mgh = Fh,$$

where U is the potential energy [J], m is the mass [kg], g is the gravitational acceleration due to the field [N/kg], h is the height of the mass from the zero-potential [m], and F is the force on the mass [N]. Dimensionally, the result of force multiplied with distance is Newton-meters [N-m], which is equivalent to Joules [J].

Electrical potential energy can be determined by:

$$U = qEs = Fs,$$

where U is the potential energy [J], q is the charge [C], E is the electrical acceleration due to the field [N/C], s is the distance of the charge from the zero-potential [m], and F is the force on the charge [N]. Again, the dimensions of the force by the distance are units of energy (Joules).

I.5 Potentials

Sometimes the electrical potential energy is also described as,

$$U = Vq,$$

where the electrical potential (voltage), V, is defined as,

$$V = Es.$$

This "charge-independent" value is known as electrical potential, or voltage, which has units of volts [V] or Joules per Coulomb [J/C]. The same can be done to gravitational potential energy to determine a "mass-independent" gravitational potential.

$$U = mgh,$$

becomes,

$$U = mV.$$

So, the gravitational potential is,

$$V = \frac{U}{m},$$

which becomes,

$$V = gh.$$

In general, the potential is the potential energy per source unit [J/kg or J/C] and represents energy density, thus does not depend on mass or charge, respectively.

II. Static System Parameters and Units

Thus, we have the following table of field parameters in gravitation and electromagnetism:

Generalized Units	Field Parameter	Gravitation	Electromagnetism
[Source]	**Source**	m **Gravitational Mass** [kg]	q **Electrical Charge** [Coulomb]
[Field] = [Force/Source]	**Field (Acceleration)**	g **Gravitic Field** [N/kg] [m/s^2]	E **Electric Field** [N/C] [V/m]
[Force] = [Source*Field]	**Force**	$F = ma$ **Gravitic Force** [N] [$kg \cdot m/s^2$]	F = qE **Electric Force** [N] [$kg \cdot m/s^2$]
[Energy] = [Force*Displacement]	**Potential Energy**	U = mgh = Fh = Vm = VF/g **Gravitational Potential Energy** [J] [N·m] [$kg \cdot m^2/s^2$]	U = qEs = Fs = Vq = VF/E **Electrical Potential Energy** [J] [N·m] [$kg \cdot m^2/s^2$]
[Potential] = [Energy/Source]	**Potential**	V = U/m = gh **Gravitational Potential** [J/kg] [m^2/s^2]	V = U/q = Es **Electrical Potential** [J/C] [V]

Drawing comparisons between the field parameters, some generalizations can be made. Most broadly, and most fundamental, is that Sources produce Fields and exert Forces. Forces are the products of Sources and Fields. The (potential) energy required to exert such forces is dependent on the displacement of the source in the field and is the product of that displacement with the exerted force. Potential is a measure of energy density and is the proportion of energy units to source units.

Dimensionally, Fields are the proportion of Force units to Source units. Energies are measured in Joules and Potentials are energy ratios dependent on the source (mass, charge). While the first four parameters can be rather intuitive (especially after comparing gravitation and electromagnetism), the fifth parameter - potential - is not as straightforward.

So then if voltages are now perhaps more "familiar", how is a gravitational potential relatable? What would be the gravitational equivalent/analog of voltage?

Recall, g [N/kg] and E [N/C] which are both written such that the units are given by the ratio of the force units to source units.

Thus, a source interacts with a field and experiences a force. The force is proportional to the magnitude of acceleration. For an arbitrary source, p, and acceleration field, A, the force experienced by the source in the field is given by

$$F = pA$$

To be more accurate, the field is produced by a parent source with which a child source interacts and experiences the force. Recall these equations of Newton and Coulomb:

$$F = -G\frac{mM}{R^2}$$

And

$$F = k\frac{qQ}{R^2}$$

Maybe more interesting is the relationship that can be drawn between units of potential. Until the units of the gravitational field were rewritten as force units per source units, they were simply [m/s^2] and identified as acceleration. When comparing units of the alternative expression for the electric field [V/m], the relationship is easily identified.

If gravitational acceleration units are rewritten as [(m^2/s^2)/m] and compared to [V/m], then Volts can be related to the units of gravitational potential as m^2/s^2. To answer a previous question, gravitational and electrical potential are both expressions of energy units per source units, and thus, gravitational potential [m^2/s^2] can be thought of as gravitational voltage (though probably shouldn't be). Perhaps it would be more appropriate to refer it to as gravitational "pressure", as voltage is sometimes likened to electrical "pressure".

But what does a m^2/s^2 represent physically? An acceleration over a distance, which is caused by a force and driven by energy. If we consider a voltage across two parallel plates, with one being grounded, there exist a series of parallel equipotentials. For example, a 1 V potential over a 1 mm distance, there will exist a potential of 0.5 V half way between the plates. At a distance of 0.25 mm from the ground plate, the voltage will be 0.25 V. Hence, the acceleration depends on the voltage gradient, or perhaps a gradient of active energy conversion – as the more energy/time available to accelerate the particle, the more energy is converted from potential energy to kinetic energy. "Storing" potential energy would consist of converting kinetic energy and the acceleration would depend on a gradient of passive energy conversion.

III. Dynamics

So far, only the static displacement of sources has been discussed. What about the movement of these sources at some velocity, v? Then,

Mass x Velocity = Momentum = [kg x m/s]

&

Charge x Velocity = [Coulomb x m/s]

So, what does this unit represent for electrodynamics?

Consider a resistive force, R, that is proportional to velocity by a constant such that

$$R = Bv$$

Let's look at the units of this constant, b,

$$\mathrm{N} = [B]\frac{\mathrm{m}}{\mathrm{s}}$$

$$[B] = \frac{\mathrm{N}\cdot\mathrm{s}}{\mathrm{m}} = \frac{\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}}{\mathrm{s}^2\cdot\mathrm{m}} = \frac{\mathrm{kg}}{\mathrm{s}}$$

Then, b is a mass flow rate. (Is there a relation of current times velocity? What is it?)

Let's now consider how this force relates to potential, where a is the net acceleration produced by the force, and h is the distance over which the force is applied.

$$V = ah = \frac{F}{m}h$$

If we substitute the force with the resistive force, R,

$$V = \frac{R}{m}h = \frac{Bv}{m}h = \frac{B}{m}hv$$

Written in the order of the last expression above, the potential is represented as the product of the ratio of mass flow rate to mass, distance, and velocity.

We should check dimensionally that this in indeed a potential,

$$\frac{\mathrm{m}^2}{\mathrm{s}^2} = \left(\frac{\frac{\mathrm{kg}}{\mathrm{s}}}{\mathrm{kg}}\right)(\mathrm{m})\left(\frac{\mathrm{m}}{\mathrm{s}}\right)$$

And we see that this does in fact agree with the dimension of potential!

Let's rearrange the expression for potential to isolate the mass flow rate,

$$V = B\left(\frac{hv}{m}\right)$$

Doing this, the term in parentheses effectively represents the "resistance" of mass flow!

$$V = BR$$

This equation is effectively a **gravitational "Ohm's Law"**, where B is the mass flow rate (mass-current), and R is the flow resistance (of which the units are strange). The units of the flow resistance will be referred to as "Gohs", for fun, (unless this unit already exists and has a better name)

$$[Gohs] = [R] = \frac{\mathrm{m}^2}{\mathrm{kg} \cdot \mathrm{s}}$$

If you stick around until the end, I'll show you how these units actually agree with the resistive unit of Ohm's, with a small (potentially hand-wavy) trick.

With a possible gravitational analogy to Ohm's law (for ohmic devices), let's turn our attention to the three passive components of electric circuits and attempt to "derive" gravitational analogies to the resistor, capacitor, and inductor.

Since we have somewhat described the resistive component, with the flow resistance R, let's start with the capacitor. Consider how a spring is like a capacitor. Both are capable of "storing" charge given that they are not overworked. Both stored energies would like to discharge and induce a force proportional to the displacement of the energy sources (mass/charge). Although Hooke's Law for springs is an approximation, if we permit that the function of these devices is analogous (and the series/parallel combinations of these devices is identical – capacitors and springs in parallel add!). In short, both springs and capacitors store energy as a function of displacement of the source.

Comparing the two general force equations to highlight their similarities,

$$F = ma \qquad F = qE$$

Starting with relation between capacitance, charge, and voltage,

$$Q = CV \qquad C = \frac{Q}{V}$$

By analogy, if H represents the mass-capacitance, we seek something of the form,

$$M = HV \qquad H = \frac{M}{V}$$

Where H has the following units:

$$\frac{\text{kg} \cdot \text{s}^2}{\text{m}^2}$$

Let's check that this quantity holds in other familiar expressions.

Recall that potential is related to the electric field and charge separation distance by,

$$V = Ed \qquad E = \frac{V}{d}$$

Then, for the Coulomb force,

$$F = qE = q\frac{V}{d}$$

And

$$V = \frac{Q}{C}$$

So, the Coulomb force could also be expressed in terms of capacitance as,

$$F = \frac{qQ}{dC}$$

Which suggests there exists an analogous Newtonian force,

$$F = \frac{mM}{dH}$$

Checking dimensionally,

$$\text{N} = \frac{\text{kg}^2}{\text{m}\left(\frac{\text{kg} \cdot \text{s}^2}{\text{m}^2}\right)} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

This checks out! How about for potential energy? The potential energy of a capacitor is given by

$$PE = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}C(Ed)^2$$

Then, by analogy, the potential energy of a mass-like capacitor is,

$$PE = \frac{1}{2}MV = \frac{1}{2}HV^2 = \frac{1}{2}H(ad)^2$$

Checking the units to verify they are units of energy,

$$\mathrm{J} = \left(\frac{\mathrm{kg} \cdot \mathrm{s}^2}{\mathrm{m}^2}\right)\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)^2 (\mathrm{m})^2 = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2} = \mathrm{N} \cdot \mathrm{m}$$

This checks out!

We could also do this for the extended expression of potential energy, $PE = \frac{1}{2}C(Ed)^2$, though this will yield the same result.

Instead, let's think about how this mass-capacitance, H, relates to the stiffness, k, of a spring. Recall that the potential energy of a spring is given by,

$$PE = \frac{1}{2}kx^2 = \frac{1}{2}HV^2$$

Solving for the stiffness,

$$k = \frac{HV^2}{x^2}$$

Let's check this dimensionally,

$$[k] = \frac{\left(\frac{\mathrm{kg} \cdot \mathrm{s}^2}{\mathrm{m}^2}\right)\left(\frac{\mathrm{m}^2}{\mathrm{s}^2}\right)}{\mathrm{m}^2} = \frac{\mathrm{kg}}{\mathrm{s}^2} = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2 \cdot \mathrm{m}} = \frac{\mathrm{N}}{\mathrm{m}}$$

This too checks out! Pretty cool!

So,

$$k = \frac{HV^2}{x^2} = \frac{F}{x}$$

Then, the mass-capacitance can be expressed in the following forms:

$$H = \frac{m}{V} = \frac{kx^2}{V^2} = \frac{mM}{Fx}$$

Now we will investigate mass inductance.

$$V_L = L\frac{dI}{dt} = L\ddot{q}$$

So, an analogous description of potential using mass induction, N

$$V_N = N\ddot{m}$$

Still not yet sure exactly what a variable for mass-current would be. Though, we can at least explore the units of mass induction,

$$N = \frac{V_N}{\ddot{m}}$$

Dimensionally,

$$\frac{\frac{\mathrm{m}^2}{\mathrm{s}^2}}{\frac{\mathrm{kg}}{\mathrm{s}^2}} = \frac{\mathrm{m}^2}{\mathrm{kg}}$$

Or, equally,

$$\frac{\mathrm{m}^2}{\mathrm{kg}} = \frac{\text{Area}}{\text{mass}}$$

How might we confirm the units of this quantity? And what does gravitational inductance look like?

Consider the following:

$$V = \frac{W}{m} = \frac{Fd}{m} = F\left(\frac{d}{m}\right)$$

Notice that the potential is the product of force and the ratio of distance to mass. Rearranging for an expression of force,

$$F = m\frac{V}{d}$$

Where now force is expressed as the product of mass and the ratio of potential "spread out" over a distance (gradient?) Let's then consider the expression of force as a gradient of potential! Recall, (for one-dimension):

$$F = \nabla\phi = \nabla V = \frac{dV}{dx}$$

Comparing these two expressions, and noting that the distance, d, will be comparative with the x coordinate in one dimension.

$$F = m\frac{V}{d} = \frac{dV}{dx}$$

If these expressions of force are to be equal, it would seem to imply that the differential of potential must possess some information about mass. This is also suggested by evaluating the statements dimensionally.

$$\mathrm{N} \neq \frac{\frac{\mathrm{m}^2}{\mathrm{s}^2}}{\mathrm{m}} = \mathrm{m/s}^2$$

The units of the potential differential over one dimension of space are acceleration units, so perhaps the expression is incorrect.

I posit that the expression might instead be, with the only premise being the necessity of a mass unit:

$$F = m\nabla\phi$$

***I will explore this matter further and return with an update to the force expression involving a gradient!**

Comparing units of mass inductance and charge inductance

$$[N] = \frac{\mathrm{m}^2}{\mathrm{kg}}$$

$$[L] = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{A}^2\cdot s^2} = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{C}^2}$$

At first glance, these may seem irreconcilable. However, if we consider a potentially hand-wavy manipulation as was mentioned earlier, this can be achieved under the following premise:

When switching between paradigms of mass and charge, let

$$\mathrm{kg} = \mathrm{C}$$

Blasphemy indeed, but it does seem to work!

$$[L] = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{C}^2} = \frac{\mathrm{kg}\cdot\mathrm{m}^2}{\mathrm{kg}^2} = \frac{\mathrm{m}^2}{\mathrm{kg}} = [N]$$

What happens when both are present is not being considered here, and hopefully I will get to that topic in due time. With this hand-waving out of the way. Let's turn our attention to another interesting relationship that can be recognized dimensionally, specifically involving time.

However, before discussing the temporal relationship of these quantities, let's explore one more relationship.

Consider the work done by a force over a distance,

$$W = Fd$$

If the units of mass-inductance are given by

$$[N] = \frac{Area}{mass}$$

How might this quantity relate to the work done by a force? Consider manipulating the expression for work as follows,

$$W = Fd = mad = mV$$

Where V is the potential in $\mathrm{m}^2/\mathrm{s}^2$. So, kinetic energy (work) is the product of mass and potential,

$$[W] = \mathrm{kg} \cdot \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

Or, rearranging,

$$[W] = \mathrm{m}^2 \cdot \frac{\mathrm{kg}}{\mathrm{s}^2}$$

Which reveals an interesting description of kinetic energy (work) as it relates to Area. Also, consider the force done by a spring as it relates to a "stiffness" of the system, k, whereby,

$$F = kd$$

So, the work done by the spring,

$$W = Fd = (kd)d = kd^2$$

Notice that if we match this expression for work with the units above,

$$W = d^2 k$$

Then the stiffness is related to mass per time squared,

$$k = \frac{m}{t^2}$$

We can also solve for the stiffness in terms of energy,

$$k = \frac{W}{d^2} = \frac{W}{A}$$

So, the dimension of stiffness is energy per area! This is interesting! You might recall that the dimension of stiffness is also expressed as force per distance (from $k = \frac{F}{d}$), so let's verify these units.

$$k = \frac{F}{d} = \frac{W}{A} = \frac{m}{t^2}$$

$$\frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} \cdot \frac{1}{\mathrm{m}} = \frac{\mathrm{kg}}{\mathrm{s}^2}$$

$$\frac{\mathrm{J}}{\mathrm{m}^2} = \frac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{s}^2} \cdot \frac{1}{\mathrm{m}^2} = \frac{\mathrm{kg}}{\mathrm{s}^2}$$

Thus, the dimension of stiffness can be described as

$$[k] = \frac{\mathrm{N}}{\mathrm{m}} = \frac{\mathrm{J}}{\mathrm{m}^2} = \frac{\mathrm{kg}}{\mathrm{s}^2}$$

And it is this last set of units which I find the most interesting and leads us nicely into the next conversation involving the dimension of time.

I now want to compare units of resistance, capacitance, and inductance as well as stiffness.

Recap,

Type equation here.

IV. Potential Energy vs Kinetic Energy

What I am struggling with comprehending/describing is HOW objects/sources obtain potential energy! And HOW that energy is converted to kinetic energy! Heaviside said that potential energy is just energy that is not known to be kinetic!

I've attempted to describe potential energy as energy obtained by a source displaced in a displacement field (gravitic/electric) and is determined by a displacement (space), while kinetic energy is energy obtained (or lost) through movement in a displacement field and is determined by velocity (time).

This prescription worked well enough for mass, as a mass raised to some height obtains an amount of potential energy determined by that height and kinetic energy is obtained by the mass as it is released and moves through the field.

On ground (Zero-potential) --> Raised to some height (Has potential) --> Dropped from some height (Loses potential - Gains momentum)

Of course, potential energy is converted (imperfectly) to kinetic energy. The concepts of inertia and momentum are essentially identical (based on Newton's first law), where an object at rest (inert) tends to stay at rest and an object in motion (having momentum) tends to stay in motion, unless acted upon by an external force.

Now we must deal with forces! We've discussed how sources generate displacement fields and an action induced by a force creates a moment on the source. However, using the potential/kinetic formulation described here fails to fully rationalize the dynamics of electromagnetism and its units.

IV.1 Gravitational Energy (Gravitoheavitic)

Consider a ball on a desk. If we draw out coordinate axes such that the top of the desk is the "zero-potential", we can still apply the laws of motion as appropriately as if we had chosen the ground to be the location of "zero-potential". Now consider the ball raised to some height. The ball now "obtains" potential energy and, when released from rest, the energy will be converted into kinetic energy (with some - often negligible - losses). Without considering the force nor the energy necessary to raise the ball, we begin by associating some finite amount of possessed potential energy.

This amount of potential energy is determined by the height the mass was raised with respect to the zero-potential, $U = mgh$, where g is the acceleration produced by the displacement field. We also know that work is done, $W = Fd$, so here, the energy needed to perform that work $U = mgh$ can be recognized as $U = Fh$ where F is the gravitational force on a mass, m, and h is the displacement height.

So, in essence, a source of mass, m, in kg, raised or displacement by some height or distance from the zero-potential, h, in a displacement field, g, obtains the potential energy U [Joules = kgm^2/s^2]. This justifies the prescription thus far.

As the ball in the example is released, potential energy is converted into kinetic energy. As the acceleration field acts on the ball, which experiences a force ($F = mg$), it begins to fall and move with an increasing velocity over time. Kinetic energy is then determined by the rate of change of position and depends on time. K = (1/2)mv^2 [units of Joules]. With this motion through the acceleration field, here a gravitational field, a cogravitational field is produced normal to the gravitational field and the direction of motion of the source. The cogravitational field is oriented such as to rotate positively about the axis of motion.

Consider that a source displaced (and sustained/maintained in the displaced position) in an acceleration field by some distance (from a zero-potential) experiences a force and possesses some amount of potential energy. Upon releasing the source from its fixation and allowing the force to act, another field is produced as the source moves through the acceleration field and potential energy is converted (imperfectly) into kinetic energy. The secondary field is a rotational field, while the primary field is rectilinear.

IV.2 Electromagnetic Energy

Consider a charge displaced in an electric field. Note the equipotentials (1 - 5V) and the dotted field lines.

A positive source charge shown experiences a force, F, due to the electric field. As described previously, a source displaced in an acceleration field experiences a force. We know that F = qE and this is strikingly similar to F = ma as the force being the product of the source (mass/charge) and the field's acceleration.

Then, we've seen that the sources and the acceleration fields, forces, and energies (potential/kinetic) experienced by a source have similar representations in both electromagnetism and gravitation.

IV.3 Generalized Energy

The acceleration field produced by a parent source has units that can also be written as the potential per distance, or in other words, the energy per source per distance. It is interesting to consider various arrangements of these components (source, field, force, distance, potential, and potential energy), but how exactly do sources "obtain" energy? Seemingly, they acquire the energy as they are displaced in an acceleration field, i.e., U = Fd = Fh = Fs. The energy is the work done by the force F to displace the object in the field.

Then, it would be reasonable to conclude that potential energy is determined by a sources displacement in an acceleration field, whose magnitude is determined by the parent source. In other words, potential energy is dependent on position in an acceleration field. Potential energy can also be defined as potential per distance.

I propose that:

Potential energy depends on position in an acceleration field.

Before exploring kinetic energy and rotational fields, I would also propose that:

Kinetic energy depends on motion in an acceleration field.

Still, this does nothing to exhaust Heaviside's statement that potential energy is energy that is not known to be kinetic.

To describe the motion of a source through a field, let's first consider how a source is set in motion. A change in state requires a force (acceleration). A source can only be accelerated by a force and a source only experiences a force when accelerated. To begin motion requires a (net) force!

Then, maybe, it would be appropriate to say that kinetic energy is the energy of a source experiencing a net force while potential energy is the energy of a source experiencing no net forces (inert)?

V. Dynamic System Parameters and Units

Generalized Units	Field Parameter	Gravitation	Electromagnetism	Economics
[Displacement]	**Displacement**	$\Delta x = x - x_0 = d = h$ [meters]	$\Delta s = s - s_0 = s$	
[Motion] = [Displacement/Time]	**Velocity**	$v = \frac{d}{t}$ [m/s]	$v = \frac{s}{t}$ [m/s]	
[Source]	**Source**	m **Gravitational Mass** [kilograms]	q **Electrical Charge** [Coulomb]	e **Money** [$]
[Acceleration] = [Field] = [Force/Source]	**Static Field (Acceleration)**	g **Gravitic Field** [N/kg] [m/s^2]	E **Electric Field** [N/C] [V/m]	C **Economic Field** [Force/$]

[Force] = [Source*Field]	**Static Force**	F = mg $F = G\frac{mM}{R^2} = \frac{mM}{Vh}$ **Gravitic Force** [N] [kg·m/s^2]	F = qE $F = k\frac{qQ}{R^2} = \frac{qQ}{Vs}$ **Electric Force** [N] [kg·m/s^2]	F = eC **Economic Force** [Force]
[Energy] = [Force*Displacement]	**Potential Energy**	U = mgh = Fh = Vm = VF/g **Gravitational Potential Energy** [J] [N·m] [kg·m^2/s^2]	U = qEs = Fs = Vq = VF/E **Electrical Potential Energy** [J] [N·m] [kg·m^2/s^2]	
[Potential] = [Energy/Source]	**Potential**	V = U/m = gh **Gravitational Potential** [J/kg] [m^2/s^2]	V = U/q = Es **Electrical Potential** [J/C] [V]	
[Field] = ~~[Force/Source]~~	**Dynamic Field**	**Heavitic Field** [] [] []	**Magnetic Field** [] []	
[Force] = ~~[Source*Field]~~	**Dynamic Force**	**Heavitic Force** [] []	**Magnetic Force** [] []	
[Energy] = [Source*Potential]	**Kinetic Energy**	K = (1/2)mv^2 **Gravitational Kinetic Energy** [J] [N·m] [kg·m^2/s^2]	**Electrical Kinetic Energy** [] []	
[Power] = [Energy/Time]	**Power**	P = W/t	P = IE = RI^2	

[Momentum] = [Source*Motion]	**Momentum**	$p = mv = Ft$ **Momentum** [kg·m/s]	~~I = nqAv ; I = q/t~~ ~~**Current**~~ ~~[Coulomb/s]~~ $J = pv$ **Current Density** [Coulomb/m^2·s]	
[Rotation]	**Torque**	$T = rF$ **Torque** [N·m]		
	Pressure	$P = F/A$ [N/m^2] [kg/m·s^2] [Joules/m^3] [Pa]		
	Flux	**Mass Density Current** [kg/m^3/s]	~~**Electron Current**~~ ~~[Coulomb/s]~~	
	Impedance			
Capacitance, H / C [Source/Potential] Stiffness, k [Energy/Area]	**Capacitance (Potential Energy)** $\boldsymbol{k = \frac{F}{x} = \frac{W}{A}}$ $\boldsymbol{k = \frac{HV^2}{x^2}}$ $\boldsymbol{k = \frac{CV^2}{x^2}}$	**Spring** $H = \frac{m}{V} = \frac{kx^2}{V^2} = \frac{mM}{Fd}$ $\left(U = \frac{1}{2}kx^2 = \frac{1}{2}HV^2\right)$ $\left(U = \frac{1}{2}Hg^2d^2\right)$ $[\frac{\text{kg}\cdot\text{s}^2}{m^2}]$	**Capacitor** $C = \frac{Q}{V} = \frac{qQ}{Fs}$ $\left(U = \frac{1}{2}QV = \frac{1}{2}CV^2\right)$	Storage of **Capital** [$ stock/ $ demand]

			$\left(U = \frac{1}{2}CE^2s^2\right)$ [Farads][Coulomb/Volt] [Coulomb^2/Joule] [s/Ohm]	
Inductance, I / L [Area/Source]	**Inductance (Kinetic Energy)**	**Flywheel (Inertia)** $I = \frac{A}{m}$	**Inductor** [Henries] [V·s/A] [Joule·s^2/Coulomb^2] [Ohm·s]	Mobility of **Services**
	Conductance/ (Energy Dissipation)	**Dashpot/Damper (Friction)**	**Conductor** [Siemens (mhos)] [A/V] [1/Ohm]	Materials for **Goods** [$/yr/demand]

Unsorted Notes:

"impedance … is defined as the ratio of pressure to volume flow rate"
https://en.wikipedia.org/wiki/Hydraulic_analogy

Impedance is the ratio of pressure to flux.

Pressure induces a flow rate proportional to impedance.

Capacitance induces inductance proportional to conductance.

A capacity of stored potential energy can induce kinetic energy proportional to the ability to conduce energy dissipation.

The three ideal passive energy components of electronics, the capacitor, the resistor, and the inductor correspond to three ideal passive energy components of economics called the pure industries of capital, goods, and services, respectively.

Economic capacitance represents the storage of capital in one form or another.

Economic conductance represents the level of conductance of materials for the production of goods.

Economic inductance represents the inertia of the economic value in motion. This is a population phenomenon known as services.

Economic Inductance

An electrical inductor (e.g., a coil of wire) has an electric current as its primary phenomenon and a magnetic field as its secondary phenomenon (inertia). Corresponding to this, an economic inductor has a flow of economic value as its primary phenomenon and a population field as its secondary phenomenon of inertia. When the flow of economic value (e.g., money) diminishes, the human population field collapses to keep the economic value (money) flowing (extreme case – war).

This public inertia is a result of consumer buying habits, expected standard of living, etc., and is generally a phenomenon of self-preservation.

Inductive Factors To Consider

(1) Population

(2) Magnitude of the economic activities of the government

(3) The method of financing these government activities (See Peter-Paul Principle – inflation of currency.)

Translation

(A few examples will be given.)

Charge – coulombs – dollars (1939).
Flow/Current – amperes (coulombs per second) – dollars of flow per year.
Motivating Force – volts – dollars (output) demand.
Conductance – amperes per volt – dollars of flow per year per dollar demand.
Capacitance – coulombs per volt – dollars of production inventory/stock per dollar demand.

From thesis:

Particles vs waves

Particles and motion

Waves and propagation

Charge and current

Mass and momentum

These two concepts seem to be at the basis of the two fields (as if related by the unsharpness principle)

Sources and Fields

Inertial mass (gravitational mass) generates a radially inward gravitational field and the forced motion of that mass (momentum) generates a rotationally positive cogravtiational field, just as an "inertial" charge (electrical charge) generates a radially outward electric field and the forced motion of that charge (current) generates a rotationally positive magnetic field.

Potential energy is the energy obtained by an object displaced (instantly) in a displacement field and is determined by a point in space. Kinetic energy is the energy of moving through a displacement field and is determined by velocity (time).

Exhaustion of potential energy from the gravitational field results in kinetic energy for the particle in proportion to the energy dissipated.