Nonlinear Human Population Growth Modeling – Upper Bias

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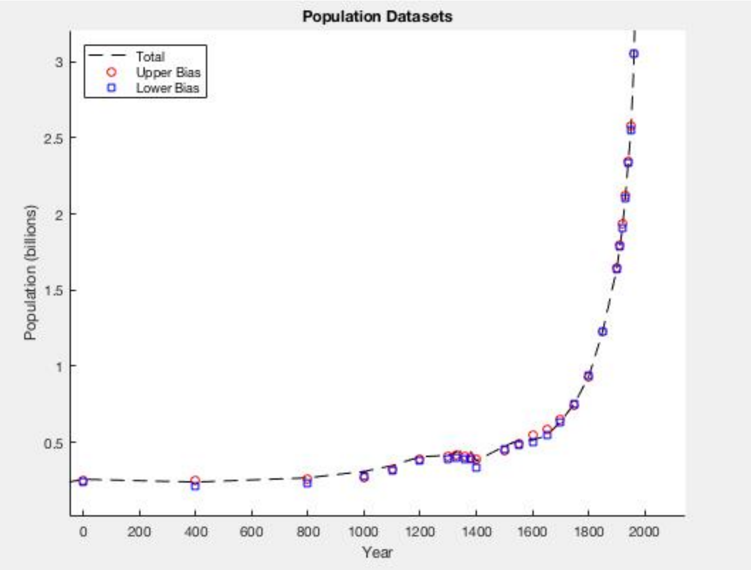
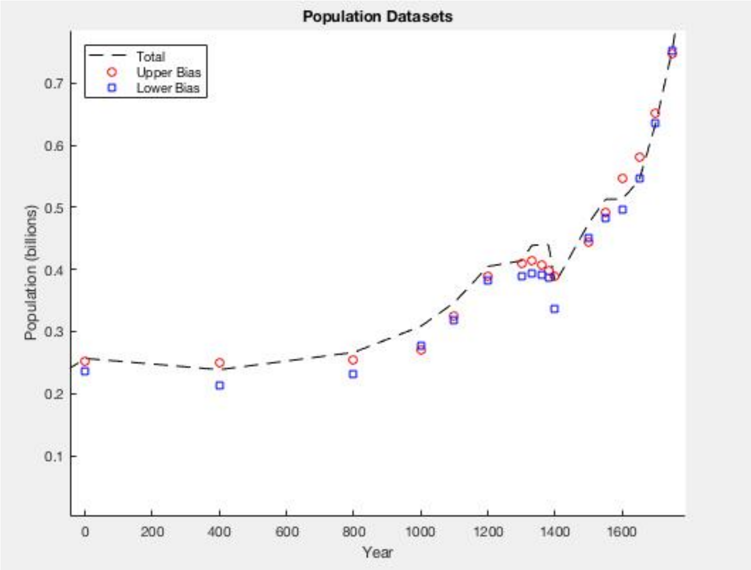
Data Compilation

- Using available census data from the USCB, UN, and other sources, a "total" was established and deemed the "canonical" dataset.
- This canonical set was compared to known historical events that significantly reduced the population:
  - Antonine Plague (165 – 180 AD)
  - Plague of Justinian (500 – 700 AD)
  - The Bubonic Plague (~ 1350 AD)

Formation of Upper Bias

- From the comparison of canonical data to the aforementioned events, some data sets were eliminated accordingly and the remaining were deemed the “upper bias”.





Models

- In order to derive a functional model of the human population over time, the Law of Mass Action and Chemical Kinetics are used to develop a relationship from known models:
  - Power
  - Logistic
  - Exponential

Power Model

- Considering the growth of population as a function of the interaction of its members leads to proportionality of the population growth to the square of the population:

$$\frac{dN}{dt} = aN^2$$

- Which has the solution:

$$\mathrm{N}(t) = \frac{N_0}{(t_o - t)^a}$$

Logistic Model

- Developed by Lotka and Voltera, the Logistic model represents the populations growth rate as proportional to the population, but assuming that the growth rate is also a function of the population:

$$\frac{dN}{dt} = a\left(1 - \frac{N}{N_0}\right)N$$

- Which has the solution:

$$N(t) = \frac{N_0}{1 + \mathrm{e}^{-a(t-t_0)}}$$

Exponential Model

- Another known model that represents the population growth as proportional to an exponential function:

$$N(t) = N_0 e^{a(t-t_0)}$$

Transition/Extinction Models

- The Power, Logistic, and Exponential models of the population were then compared to other models:
  - Stabilization
  - Lambda
  - Dielectric
  - Extinction

Parameter Optimization

- Using Excel's Solver tool, the parameters of each model were optimized by a minimization routine on the sum of squares.
- Error estimates for each parameter were also calculated.

Model Comparison

- After completing the optimization for both a full data set (408 entries) and a recapitulated data set (41 entries), the sum of squares and R-squared values were compared for each of the models.

Model Comparison

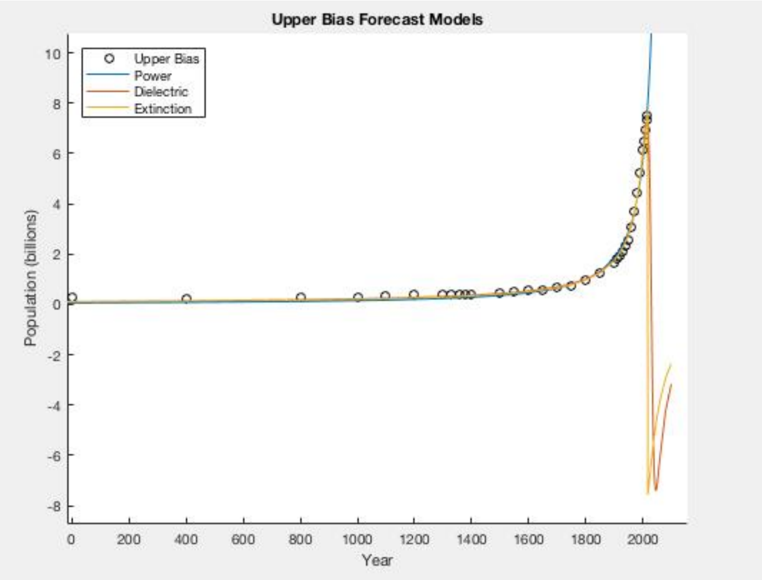
Curve Fit Parameters		Full Data Set (408 data points)				Recapitulated Data Set (41 data points)			
		Total	Error	Prather	Error	Total	Error	Prather	Error
Power	R-squared	0.99156946		0.99321966		0.99274689		0.99354517	
	SoS	14.086338	0.18603808	11.3643459	0.16709945	1.6547329	0.20339204	1.475406	0.19450173
	tt	2078.5528	0.92916868	2078.74608	0.83848212	2081.73174	2.58572688	2082.09272	10.2546555
	n	2945.38	47.0987349	2945.38332	42.3552355	2945.37436	139.02325	2945.37262	2334.04985
	a/tau	1.4286		1.42799547		1.42044767		1.41860558	0.13906023
Exponential	R-squared	0.98874062		0.98964406		0.9902565		0.99089983	
	SoS	31.3553019	0.2775609	28.6841182	0.26547495	4.28591421	0.32733447	4.09108245	0.31980785
	tt	2017.0001	0	2017.00001	0	2017.00001		2017.00001	85847766.2
	n	7.569	0.04748203	7.53796842	0.04522914	7.51991416		7.51328611	8986086.99
	a/tau	0.01439		0.01425657		0.01406331		0.01393194	
Logistic	R-squared	0.98480863		0.98541953		0.98691214		0.98739177	
	SoS	39.2585434	0.31057743	37.7022834	0.30435933	5.62795909	0.37509862	5.46175233	0.36951835
	tt	2017.00001	3.22283916	2017.00001	3.11805651	2017.00001	9.71119024	2017.00001	9.64385955
	n	14.402	0.61621977	14.6112422	0.6035584	14.6250364	1.76498551	14.6112422	1.73521703
	a/tau	0.02046		0.02041795		0.02060586		0.02041795	
Stabilization	R-squared	0.98821144		0.98980351		0.98893733		0.98969267	
	SoS	15.8936214	0.19761242	13.8259686	0.18431071	2.40566443	0.24523787	2.2615943	0.23778111
	tt	2050.7081	0.62673659	2050.78822	0.58648749	2053.74093	1.9735387	2053.99024	1.92450485
	n	286.135272	3.46718427	286.498398	3.24175332	295.063542	12.2881138	297.038826	11.9818
	a/tau	5570.61351		7131.67026		10136.0692		10136.0689	
Lambda	R-squared	0.98819273		0.99857511		0.98891763		0.98967165	
	SoS	15.90055	0.19765548	2.91904745	0.08468829	2.40459707	0.24518346	2.26081978	0.23774039
	tt	2050.747	0.62820049	2018.24738	0.40174212	2053.74285	1.97364402	2054.00332	1.9252529
	n	286.383	3.47529194	202.903708	1.11752817	295.079701	12.2893223	297.135358	11.9866323
	a/tau	0.00399		27.718717		2.9572E-05		1.0032E-05	
Dielectric	R-squared	0.99634427		0.99747235		0.99734214		0.99789509	
	SoS	5.56967752	0.11698167	3.82574699	0.09695293	0.57614326	0.12001492	0.45639826	0.10817818
	tt	2028.94115	0.28270564	2029.12412	0.24147056	2030.43858	0.62259463	2030.6035	0.8594632
	n	226.877539	0.9523888	227.64651	0.81081997	229.374424	1.94281945	231.173462	4.87627993
	a/tau	15.5344514		15.5893409		15.4797372		15.5999877	0.36255406
Extinction	R-squared	0.99600926		0.99717717		0.99701889		0.99759394	
	SoS	6.33632468	0.12477326	4.48750319	0.10500384	0.66687075	0.1291192	0.53994521	0.11766371
	tt	2017.00001	0.00083675	2017.00001	0.00070536	2017.00001	0.00095834	2017.00001	0.00096998
	n	209.560034	0.59607358	209.926331	0.50263258	210.485436	1.62758786	212.004032	3.76578283
	a/tau	41.8943102		42.0401598		42.9652131		43.2914534	1.05845462

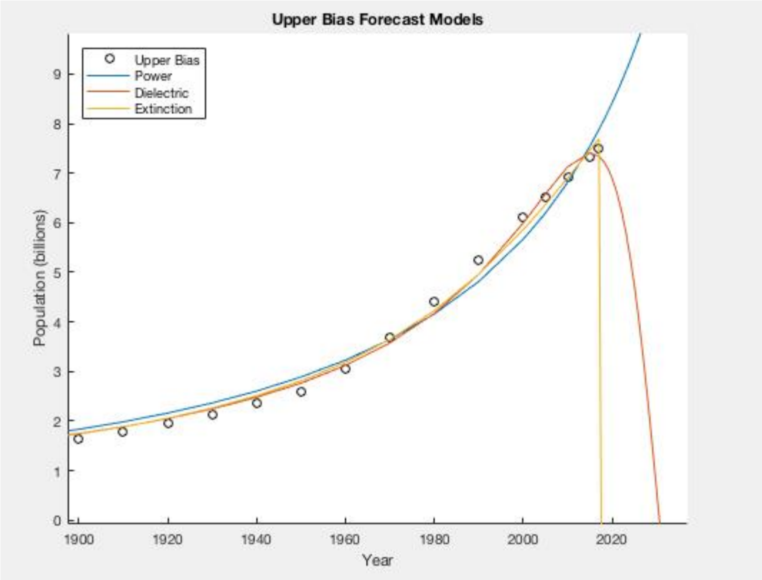
Comparison Conclusions

- Generally, the R-squared value improved for the recapitulated data.
- The Power model was a better fit than both the Logistic and Exponential models.
- The Dielectric and Extinction models had consistently high R-squared values for both data sets.

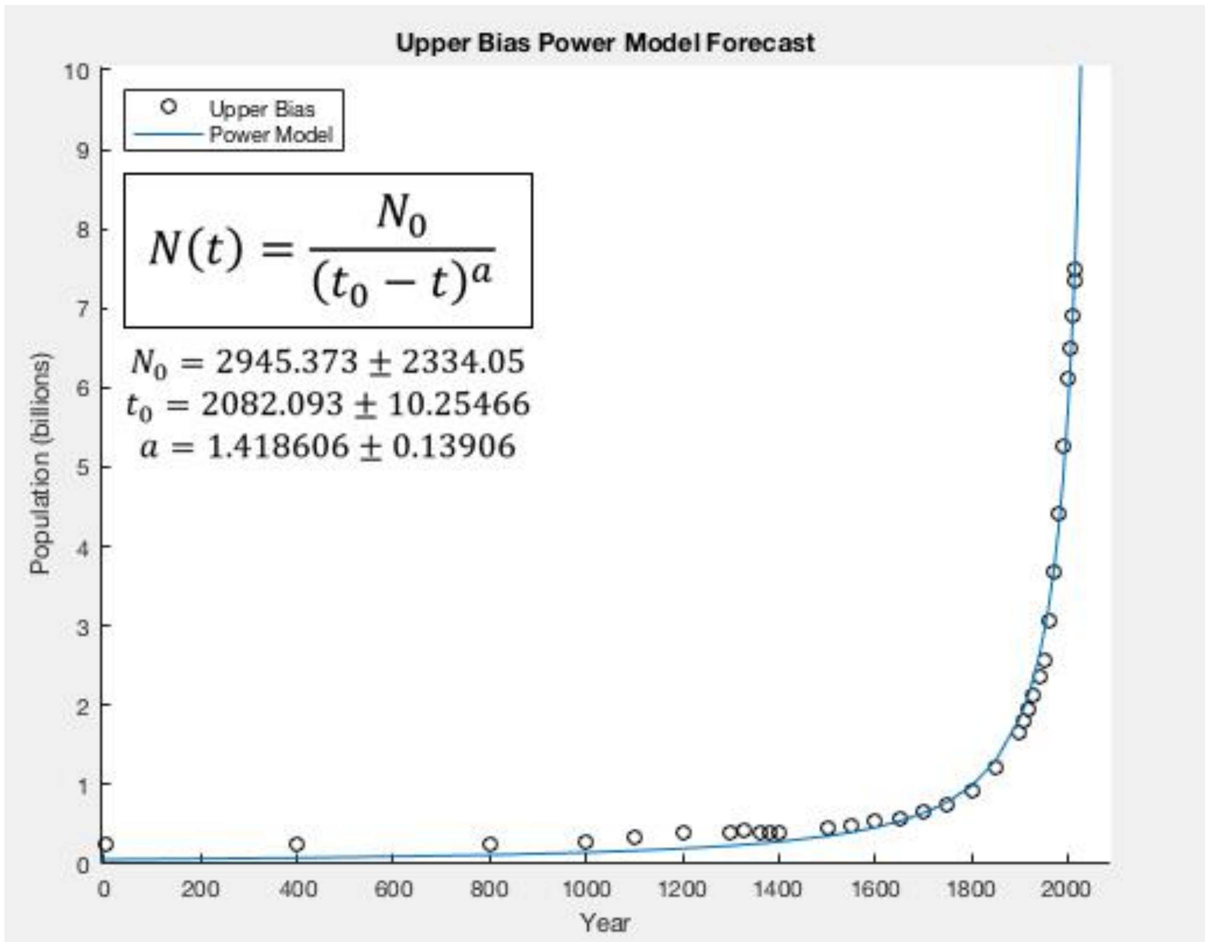
Forecasting

- Having determined the best models, the optimized parameters of each were used to extrapolate forecasts.

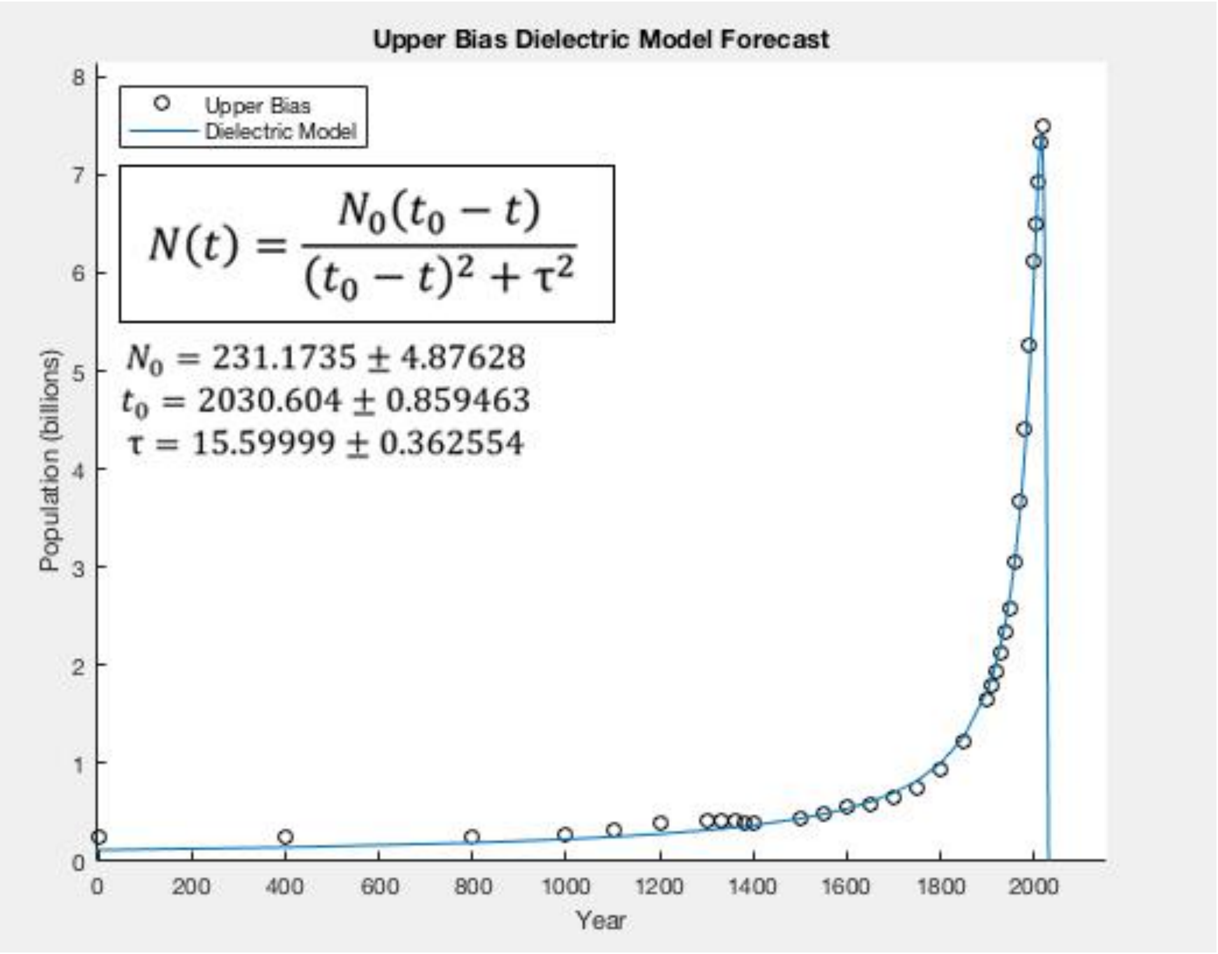




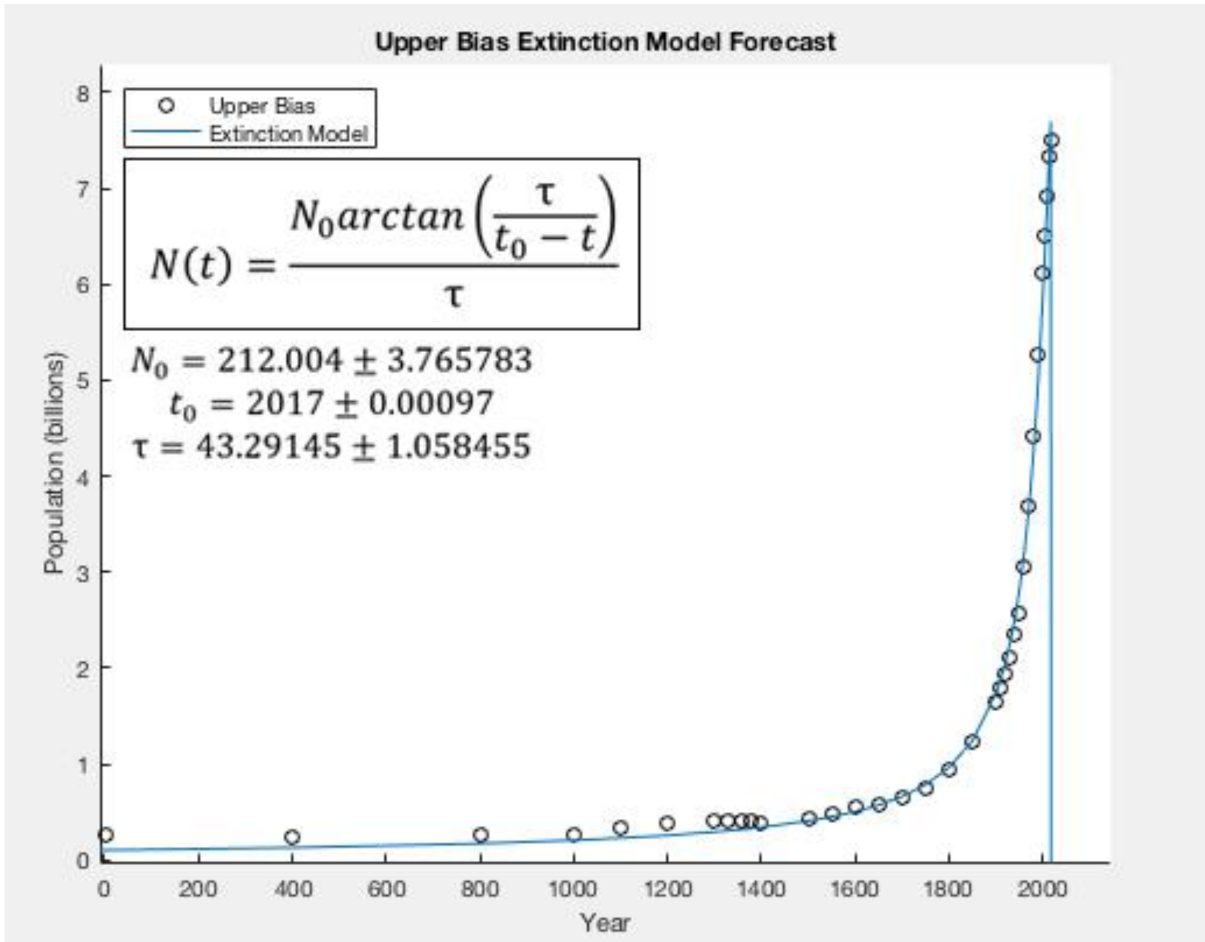
Power Model Forecast



Dielectric Model Forecast



Extinction Model Forecast



Preliminary Conclusions

- The power model has a large error associated with one of its parameters, and although having a high R-squared value, it is unlikely that the population will continue increasing indefinitely.
- The Extinction model reaches a critical point at the last known input (2017) thus is not likely an accurate prediction.
- The Dielectric model remains the most likely description of human population growth.

Citations

- Literature:

  - 1. de Levie, Robert. “Estimating Parameter Precision in Nonlinear Least Squares with Excel's Solver.” ACS Publications, Journal of Chemical Education, 11 Nov. 1999, pubs.acs.org/doi/pdf/10.1021/ed076p1594.
  - 2. Harris, Daniel C. “Nonlinear Least-Squares Curve Fitting with Microsoft Excel Solver.” ACS Publications, Journal of Chemical Education, 1 Jan. 1998, pubs.acs.org/doi/pdf/10.1021/ed075p119
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  - 4. Péter Érdi; János Tóth (1989). Mathematical Models of Chemical Reactions: Theory and Applications of Deterministic and Stochastic Models. Manchester University Press. p. 3. ISBN 978-0-7190-2208-1.
  - 5. Pielou, Evelyn C. "An introduction to mathematical ecology." An introduction to mathematical ecology. (1969).
  - 6. Nonlinear Saturation Model of World Population Growth, Reza Mofid and Weldon J. Wilson 2010
  - 7. Nonlinear Models of World Population Growth, Alan Harris and Weldon J. Wilson 2009
  - 8. A Predator-Prey Model of World Population Growth; Weldon J. Wilson 2006

- Data Sources:

  - www.aae.wisc.edu/aae641/Notes/World_Population.docx
  - https://www.census.gov/population/international/data/worldpop/table_history.php
  - http://www.scottmanning.com/archives/World%20Population%20Estimates%20Interpolated%20and%20Averaged.pdf
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  - http://data.worldbank.org/indicator/SP.POP.TOTL?end=2015&start=2015&view=map&year=1960

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