Gravitational Radiation: Maxwell-Heaviside Formulation

Author: Cole Prather

Mentor: Dr. Weldon Wilson

Outline

- Introduction
- Methodology
- Results
  - Field Equations, Potentials, and Wave Equations
  - Retarded Potentials and Fields
  - Liénard–Wiechert Potentials and Fields
  - *Lorentz Force
  - Poynting Vector
  - Gravitational Fields
  - Gravitational Radiation
- Conclusion and Future Work

Introduction

- Why reformulate gravity?

Introduction

- Why reformulate gravity?
- Why not?

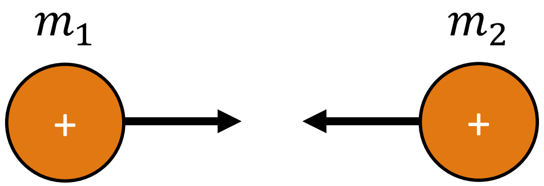
Introduction (continued)

Coulomb's Force Law

$$\boldsymbol{F} = k\,\frac{q_1 q_2}{R^2}\,\hat{\boldsymbol{R}}$$

Newton's Force Law

$$\boldsymbol{F} = -G\,\frac{m_1 m_2}{R^2}\,\hat{\boldsymbol{R}}$$



Introduction (continued)

- Gravity as an analogy to electromagnetism
  - James Maxwell
  - Oliver Heaviside
- Gravitational fields and waves
- Negative field energy
- Radiation

- What about general relativity?
  - Cannot localize gravity

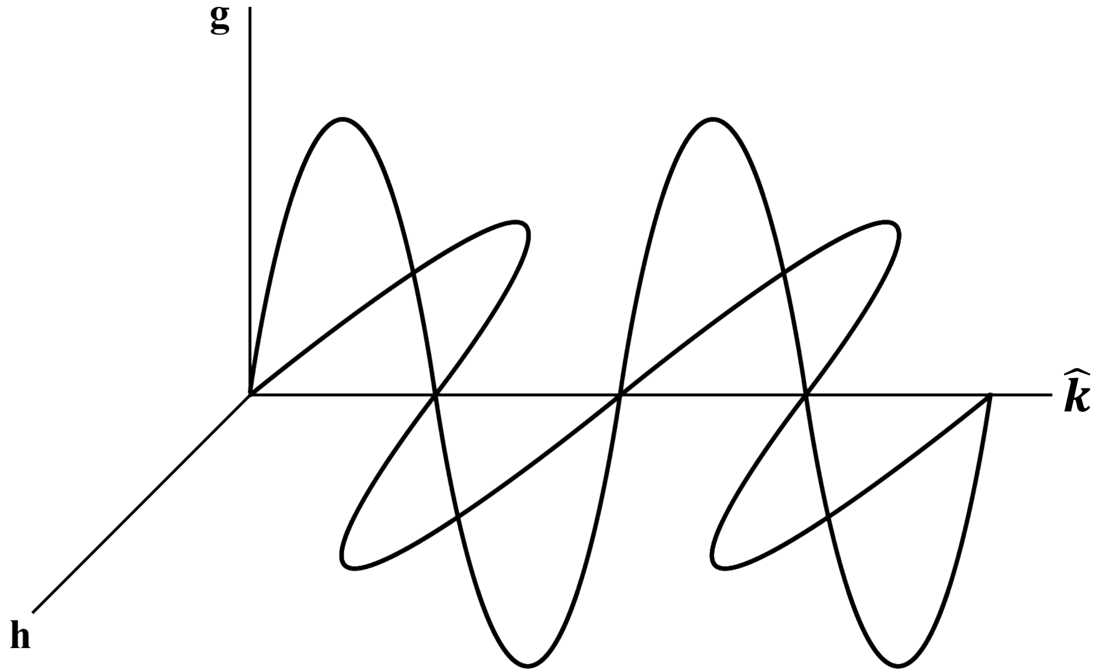


Figure 1: A gravitational wave.

Introduction (continued)

- Aristotelian (gravity)
- Ptolemaic (geocentrism)
- Copernican (heliocentrism w/ nested circles)
- Keplerian (eccentric orbits)
- Galilean (tides and phase cycles)
- Newtonian (forces)

- Einsteinian (spacetime curvature)

Introduction (continued)

- Aristotelian (gravity)
- Ptolemaic (geocentrism)
- Copernican (heliocentrism w/ nested circles)
- Keplerian (eccentric orbits)
- Galilean (tides and phase cycles)
- Newtonian (forces)
- Maxwell-Heaviside? (fields)
- Einsteinian (spacetime curvature)

Methodology

- ... By analogy!
- Field equations → Scalar and vector potentials → Wave equations
- Retarded wave propagation → Retarded potentials and fields
- Retarded potentials → Liénard–Wiechert potentials and fields
- Poynting Vector + Liénard–Wiechert Fields → Radiation
- Larmor formula for power radiated

Methodology (continued)

- Field equations → Scalar and vector potentials

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho_e$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c_e}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c_e}(4\pi \boldsymbol{J_e} + \frac{\partial \boldsymbol{E}}{\partial t})$$

Methodology (continued)

- Field equations → Scalar and vector potentials

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho_e$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c_e}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c_e}(4\pi \boldsymbol{J_e} + \frac{\partial \boldsymbol{E}}{\partial t})$$

→

Define:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A_e}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \nabla \cdot (\nabla \times \boldsymbol{A_e}) = 0$$

$$\nabla \times \nabla\phi_e = 0$$

Methodology (continued)

- Field equations → Scalar and vector potentials

$$\nabla \cdot \boldsymbol{E} = 4\pi\rho_e$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c_e}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c_e}(4\pi \boldsymbol{J_e} + \frac{\partial \boldsymbol{E}}{\partial t})$$

→

Define:

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}_e$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \nabla \cdot (\nabla \times \boldsymbol{A}_e) = 0$$

$$\nabla \times \nabla\phi_e = 0$$

→

$$\boldsymbol{E} = -\nabla\phi_e - \frac{1}{c_e}\frac{\partial \boldsymbol{A}_e}{\partial t}$$

Methodology (continued)

- Scalar and vector potentials → Wave equations

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}_e$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \nabla \cdot (\nabla \times \boldsymbol{A}_e) = 0$$

$$\nabla \times \nabla \phi_e = 0$$

$$\boldsymbol{E} = -\nabla \phi_e - \frac{1}{c_e}\frac{\partial \boldsymbol{A}_e}{\partial t}$$

Methodology (continued)

- Scalar and vector potentials → Wave equations

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}_e$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \nabla \cdot (\nabla \times \boldsymbol{A}_e) = 0$$

$$\nabla \times \nabla \phi_e = 0$$

$$\boldsymbol{E} = -\nabla \phi_e - \frac{1}{c_e}\frac{\partial \boldsymbol{A}_e}{\partial t}$$

→

Back into:

$$\nabla \cdot \boldsymbol{E} = 4\pi \rho_e$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c_e}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c_e}(4\pi \boldsymbol{J_e} + \frac{\partial \boldsymbol{E}}{\partial t})$$

Methodology (continued)

- Scalar and vector potentials → Wave equations

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}_e$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \nabla \cdot (\nabla \times \boldsymbol{A}_e) = 0$$

$$\nabla \times \nabla \phi_e = 0$$

$$\boldsymbol{E} = -\nabla \phi_e - \frac{1}{c_e}\frac{\partial \boldsymbol{A}_e}{\partial t}$$

→

Back into:

$$\nabla \cdot \boldsymbol{E} = 4\pi \rho_e$$

$$\nabla \times \boldsymbol{E} = -\frac{1}{c_e}\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{B} = \frac{1}{c_e}(4\pi \boldsymbol{J_e} + \frac{\partial \boldsymbol{E}}{\partial t})$$

→

$$\nabla^2 \boldsymbol{A_e} - \frac{1}{c_e^2}\frac{\partial^2 \boldsymbol{A_e}}{\partial t^2} = -\frac{4\pi}{c_e}\boldsymbol{J_e}$$

$$\nabla^2 \phi_e - \frac{1}{c_e^2}\frac{\partial^2 \phi_e}{\partial t^2} = -4\pi \rho_e$$

$$\nabla^2 \boldsymbol{E} - \frac{1}{c_e^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 4\pi(\nabla \rho_e + \frac{1}{c_e^2}\frac{\partial \boldsymbol{J_e}}{\partial t})$$

$$\nabla^2 \boldsymbol{B} - \frac{1}{c_e^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = -\frac{4\pi}{c_e}(\nabla \times \boldsymbol{J_e})$$

Methodology (continued)

- Retarded wave propagation → retarded potentials and fields

$$\boldsymbol{A_e}(\boldsymbol{r},t) = \frac{1}{c_e}\iiint \frac{\boldsymbol{J_e}(\boldsymbol{r}',t-\frac{R}{c_e})}{R}\,dV'$$

$$\phi_e(\boldsymbol{r},t) = \iiint \frac{\rho_e(\boldsymbol{r}',t-\frac{R}{c_e})}{R}\,dV'$$

$$\boldsymbol{E}(\boldsymbol{r},t) = \iiint \left(\frac{[\rho_e]\hat{\boldsymbol{e}}_R}{R^2} + \frac{\left[\frac{\partial \rho_e}{\partial t}\right]\hat{\boldsymbol{e}}_R}{c_e R} - \frac{\left[\frac{\partial \boldsymbol{J_e}}{\partial t}\right]}{c_e^2 R}\right) dV'$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \frac{1}{c_e}\iiint \left(\frac{[\boldsymbol{J_e}]\times\hat{\boldsymbol{e}}_R}{R^2} + \frac{\left[\frac{\partial \boldsymbol{J_e}}{\partial t}\right]\times\hat{\boldsymbol{e}}_R}{c_e R}\right) dV'$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}'$$

$$t' = t - \frac{R}{c_e}$$

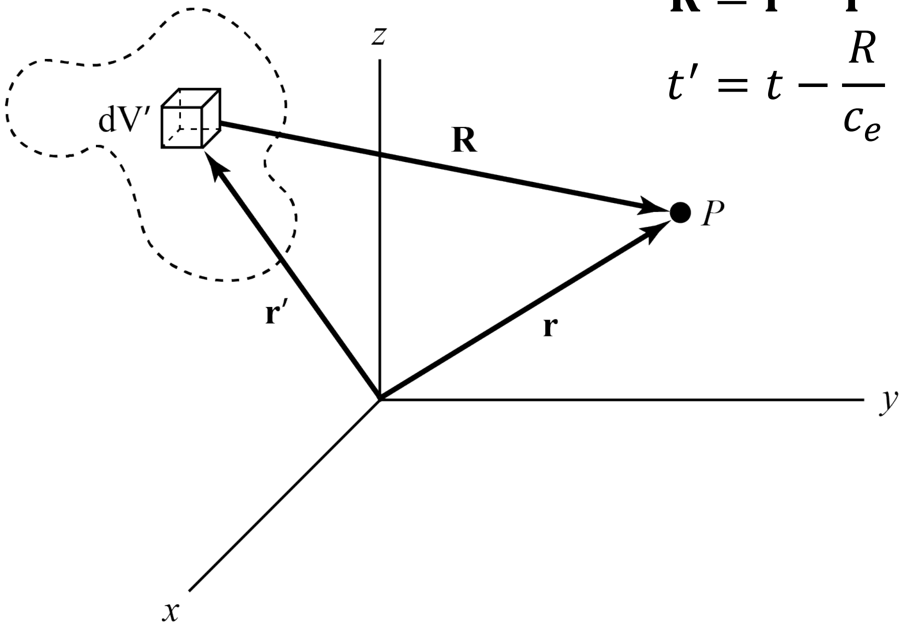


Figure 2: Delay from Retardation.

Methodology (continued)

- Retarded potentials → Liénard–Wiechert potentials and fields

$$\phi_e(\boldsymbol{r},t) = \left[\frac{e}{R(1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})}\right]\Bigg|_{t'=t-\frac{R}{c_e}}$$

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c_e}$$

$$\boldsymbol{A_e}(\boldsymbol{r},t) = \boldsymbol{\beta}\phi_e(\boldsymbol{r},t)$$

$$\boldsymbol{A_e}(\boldsymbol{r},t) = \left[\frac{e\boldsymbol{\beta}}{R(1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})}\right]\Bigg|_{t'=t-\frac{R}{c_e}}$$

$$K = (1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})$$

$$\boldsymbol{E}(\boldsymbol{r},t) = e\left[\frac{(\widehat{\boldsymbol{R}}-\boldsymbol{\beta})(1-\beta^2)}{K^3R^2} + \frac{\widehat{\boldsymbol{R}}\times\left((\widehat{\boldsymbol{R}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right)}{c_eK^3R}\right]$$

$$\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_v + \boldsymbol{E}_a$$

$$\boldsymbol{B}(\boldsymbol{r},t) = e\left[\frac{(\boldsymbol{\beta}\times\widehat{\boldsymbol{R}})(1-\beta^2)}{K^3R^2} + \frac{(\dot{\boldsymbol{\beta}}\cdot\widehat{\boldsymbol{R}})(\boldsymbol{\beta}\times\widehat{\boldsymbol{R}})}{c_eK^3R} + \frac{\dot{\boldsymbol{\beta}}\times\widehat{\boldsymbol{R}}}{c_eK^2R}\right]$$

$$\boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{B}_v + \boldsymbol{B}_a$$

$$\boldsymbol{B}_v = \boldsymbol{\beta}\times\boldsymbol{E}_v \qquad \& \qquad \boldsymbol{B}_a = \widehat{\boldsymbol{R}}\times\boldsymbol{E}_a$$

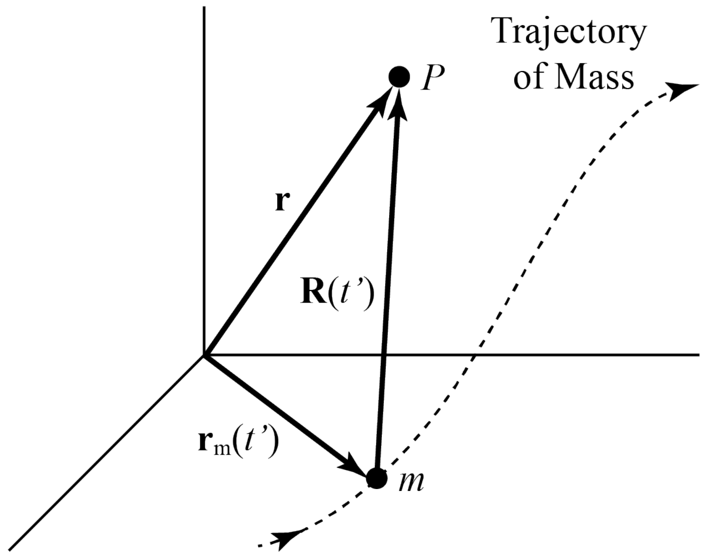


Figure 3: Geometry of a Moving Point Mass.

Methodology (continued)

The Poynting vector represents energy flux and is given by:

$$\boldsymbol{S} = \frac{c_e}{4\pi}\boldsymbol{E} \times \boldsymbol{B}$$

From the Poynting theorem:

$$\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \boldsymbol{S} + \boldsymbol{g} \cdot \boldsymbol{J} = \boldsymbol{0}$$

The velocity fields vary as an inverse-square of distance,

$$\boldsymbol{E}_{\boldsymbol{v}}, \boldsymbol{B}_{\boldsymbol{v}} \propto \frac{1}{[R^2]}$$

The acceleration fields vary as an inverse-first-power of distance,

$$\boldsymbol{E}_{\boldsymbol{a}}, \boldsymbol{B}_{\boldsymbol{a}} \propto \frac{1}{[R]}$$

So, from the following Poynting vector arrangements,

$$\boldsymbol{S}_{\boldsymbol{vv}} \propto \frac{1}{[R^4]}$$

$$\boldsymbol{S}_{\boldsymbol{aa}} \propto \frac{1}{[R^2]}$$

only accelerated charges can radiate!

Methodology (continued)

Total radiated power can be evaluated as using the Larmor formulae:

$$\frac{dP}{d\Omega} = \left(\boldsymbol{S_a} \cdot \widehat{\boldsymbol{R}}\right) R^2$$

$$P = \int_{4\pi} \frac{dP}{d\Omega} d\Omega$$

Methodology (continued)

- Retarded fields → Present fields

From the figure,

$$\boldsymbol{R}_r = \boldsymbol{R_p} + (t - t_r)\boldsymbol{v}$$

where,

$$R_r = (t - t_r)c_e$$

Leads to

$$\boldsymbol{R_p} = R_r(\widehat{\boldsymbol{R}}_{\boldsymbol{r}} - \boldsymbol{\beta})$$

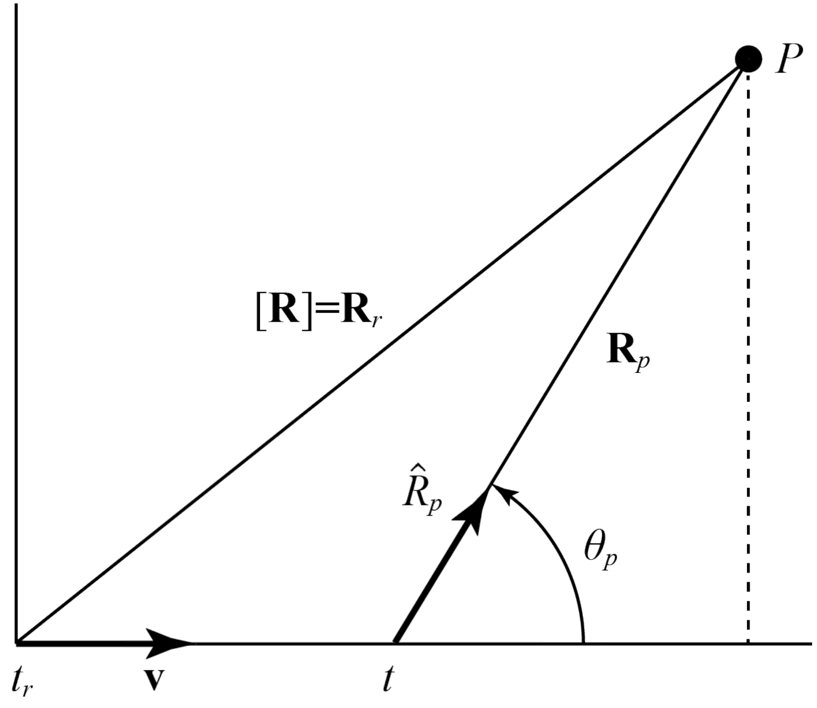


Figure 4: Retarded Position Diagram.

Field Equations

Electromagnetic Field Equations	Gravitational Field Equations
$\nabla \cdot \boldsymbol{E} = 4\pi\rho_e$	$\nabla \cdot \boldsymbol{g} = -4\pi G\rho_g$
$\nabla \times \boldsymbol{E} = -\frac{1}{c_e}\frac{\partial \boldsymbol{B}}{\partial t}$	$\nabla \times \boldsymbol{g} = -\frac{1}{c_g}\frac{\partial \boldsymbol{h}}{\partial t}$
$\nabla \cdot \boldsymbol{B} = 0$	$\nabla \cdot \boldsymbol{h} = 0$
$\nabla \times \boldsymbol{B} = \frac{1}{c_e}(4\pi \boldsymbol{J}_{\boldsymbol{e}} + \frac{\partial \boldsymbol{E}}{\partial t})$	$\nabla \times \boldsymbol{h} = \frac{1}{c_g}(-4\pi G\boldsymbol{J}_{\boldsymbol{g}} + \frac{\partial \boldsymbol{g}}{\partial t})$

Vector and Scalar Potentials

Define Electromagnetic Potentials	Define Gravitational Potentials
$\boldsymbol{B} = \nabla \times \boldsymbol{A}_e$	$\boldsymbol{h} = \nabla \times \boldsymbol{A}_g$
$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \nabla \cdot (\nabla \times \boldsymbol{A}_e) = 0$	$\boldsymbol{\nabla} \cdot \boldsymbol{h} = \nabla \cdot \left(\nabla \times \boldsymbol{A}_g\right) = 0$
$\nabla \times \nabla \phi_e = 0$	$\nabla \times \nabla \phi_g = 0$
$\boldsymbol{E} = -\nabla \phi_e - \frac{1}{c_e} \frac{\partial \boldsymbol{A}_e}{\partial t}$	$\boldsymbol{g} = -\nabla \phi_g - \frac{1}{c_g} \frac{\partial \boldsymbol{A}_g}{\partial t}$

Wave Equations

Electromagnetic Wave Equations	Gravitational Wave Equations
$\nabla^2 \boldsymbol{A_e} - \frac{1}{c_e^2}\frac{\partial^2 \boldsymbol{A_e}}{\partial t^2} = -\frac{4\pi}{c_e}\boldsymbol{J_e}$	$\nabla^2 \boldsymbol{A_g} - \frac{1}{c_g^2}\frac{\partial^2 \boldsymbol{A_g}}{\partial t^2} = \frac{4\pi G}{c_g}\boldsymbol{J_g}$
$\nabla^2 \phi_e - \frac{1}{c_e^2}\frac{\partial^2 \phi_e}{\partial t^2} = -4\pi\rho_e$	$\nabla^2 \phi_g - \frac{1}{c_g^2}\frac{\partial^2 \phi_g}{\partial t^2} = 4\pi G\rho_g$
$\nabla^2 \boldsymbol{E} - \frac{1}{c_e^2}\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 4\pi(\nabla\rho_e + \frac{1}{c_e^2}\frac{\partial \boldsymbol{J_e}}{\partial t})$	$\nabla^2 \boldsymbol{g} - \frac{1}{c_g^2}\frac{\partial^2 \boldsymbol{g}}{\partial t^2} = -4\pi G(\nabla\rho_g + \frac{1}{c_g^2}\frac{\partial \boldsymbol{J_g}}{\partial t})$
$\nabla^2 \boldsymbol{B} - \frac{1}{c_e^2}\frac{\partial^2 \boldsymbol{B}}{\partial t^2} = -\frac{4\pi}{c_e}(\nabla \times \boldsymbol{J_e})$	$\nabla^2 \boldsymbol{h} - \frac{1}{c_g^2}\frac{\partial^2 \boldsymbol{h}}{\partial t^2} = \frac{4\pi G}{c_g}(\nabla \times \boldsymbol{J_g})$

Retarded Potentials

Retarded EM Potentials	Retarded Gravitational Potentials
$\boldsymbol{A_e}(\boldsymbol{r},t) = \frac{1}{c_e}\iiint \frac{\boldsymbol{J_e}(\boldsymbol{r}',t-\frac{R}{c_e})}{R}dV'$	$\boldsymbol{A_g}(\boldsymbol{r},t) = -\frac{G}{c_g}\iiint \frac{\boldsymbol{J_g}\left(\boldsymbol{r}',t-\frac{R}{c_g}\right)}{R}dV'$
$\phi_e(\boldsymbol{r},t) = \iiint \frac{\rho_e(\boldsymbol{r}',t-\frac{R}{c_e})}{R}dV'$	$\phi_g(\boldsymbol{r},t) = -G\iiint \frac{\rho_g(\boldsymbol{r}',t-\frac{R}{c_g})}{R}dV'$

Retarded Fields

Retarded EM Fields	Retarded Gravitational Fields
$\boldsymbol{E}(\boldsymbol{r},t) = \iiint \left(\frac{[\rho_e]\hat{\boldsymbol{e}}_R}{R^2} + \frac{\left[\frac{\partial \rho_e}{\partial t}\right]\hat{\boldsymbol{e}}_R}{c_e R} - \frac{\left[\frac{\partial \boldsymbol{J_e}}{\partial t}\right]}{c_e^2 R} \right) dV'$	$\boldsymbol{g}(\boldsymbol{r},t) = -G \iiint \left(\frac{[\rho_g]\hat{\boldsymbol{e}}_R}{R^2} + \frac{\left[\frac{\partial \rho_g}{\partial t}\right]\hat{\boldsymbol{e}}_R}{c_g R} - \frac{\left[\frac{\partial \boldsymbol{J_g}}{\partial t}\right]}{c_g^2 R} \right) dV'$
$\boldsymbol{B}(\boldsymbol{r},t) = \frac{1}{c_e} \iiint \left(\frac{[\boldsymbol{J_e}] \times \hat{\boldsymbol{e}}_R}{R^2} + \frac{\left[\frac{\partial \boldsymbol{J_e}}{\partial t}\right] \times \hat{\boldsymbol{e}}_R}{c_e R} \right) dV'$	$\boldsymbol{h}(\boldsymbol{r},t) = -\frac{G}{c_g} \iiint \left(\frac{[\boldsymbol{J_g}] \times \hat{\boldsymbol{e}}_R}{R^2} + \frac{\left[\frac{\partial \boldsymbol{J_g}}{\partial t}\right] \times \hat{\boldsymbol{e}}_R}{c_g R} \right) dV'$

Oleg Jefimenko (~ 1960s) & Jose Heras (1994, 2007)

Liénard–Wiechert Potentials

… for Electromagnetism	… for Gravitation
$\phi_e(\boldsymbol{r},t) = \left[\frac{e}{R(1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})}\right]\Big\rvert_{t'=t-\frac{R}{c_e}}$	$\phi_g(\boldsymbol{r},t) = \left[-\frac{Gm}{R(1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})}\right]\Big\rvert_{t'=t-\frac{R}{c_g}}$
$\boldsymbol{A_e}(\boldsymbol{r},t) = \left[\frac{e\boldsymbol{\beta}}{R(1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})}\right]\Big\rvert_{t'=t-\frac{R}{c_e}}$	$\boldsymbol{A_g}(\boldsymbol{r},t) = \left[-\frac{Gm\boldsymbol{\beta}}{R(1-\widehat{\boldsymbol{R}}\cdot\boldsymbol{\beta})}\right]\Big\rvert_{t'=t-\frac{R}{c_g}}$

$$\boldsymbol{A_i}(\boldsymbol{r},t) = \boldsymbol{\beta}\phi_i(\boldsymbol{r},t)$$

Liénard–Wiechert Fields

… for Electromagnetism	… for Gravitation
$\boldsymbol{E} = e\left[\frac{(\widehat{\boldsymbol{R}} - \boldsymbol{\beta})(1-\beta^2)}{K^3R^2} + \frac{\widehat{\boldsymbol{R}} \times \left((\widehat{\boldsymbol{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right)}{c_e K^3 R}\right]$	$\boldsymbol{g} = -Gm\left[\frac{(\widehat{\boldsymbol{R}} - \boldsymbol{\beta})(1-\beta^2)}{K^3R^2} + \frac{\widehat{\boldsymbol{R}} \times \left((\widehat{\boldsymbol{R}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\right)}{c_g K^3 R}\right]$
$\boldsymbol{B} = e\left[\frac{(\boldsymbol{\beta} \times \widehat{\boldsymbol{R}})(1-\beta^2)}{K^3R^2} + \frac{(\dot{\boldsymbol{\beta}} \cdot \widehat{\boldsymbol{R}})(\boldsymbol{\beta} \times \widehat{\boldsymbol{R}})}{c_e K^3 R} + \frac{\dot{\boldsymbol{\beta}} \times \widehat{\boldsymbol{R}}}{c_e K^2 R}\right]$	$\boldsymbol{h} = -Gm\left[\frac{(\boldsymbol{\beta} \times \widehat{\boldsymbol{R}})(1-\beta^2)}{K^3R^2} + \frac{(\dot{\boldsymbol{\beta}} \cdot \widehat{\boldsymbol{R}})(\boldsymbol{\beta} \times \widehat{\boldsymbol{R}})}{c_g K^3 R} + \frac{\dot{\boldsymbol{\beta}} \times \widehat{\boldsymbol{R}}}{c_g K^2 R}\right]$

$$K = (1 - \widehat{\boldsymbol{R}} \cdot \boldsymbol{\beta})$$

*Lorentz Force

… for Electromagnetism	… for Gravitation
$\boldsymbol{F} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c_e} \times \boldsymbol{B}\right)$	$\boldsymbol{F} = m\left(\boldsymbol{g} + \frac{\boldsymbol{v}}{c_g} \times \boldsymbol{h}\right)$

*Lorentz Force

... for Electromagnetism	... for Gravitation
$\boldsymbol{F} = q\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c_e} \times \boldsymbol{B}\right)$	$\boldsymbol{F} = m\left(\boldsymbol{g} + \frac{\boldsymbol{v}}{c_g} \times \boldsymbol{h}\right)$

Let,

$$\boldsymbol{h} = 2c_g\boldsymbol{\omega}$$

Then,

$$\boldsymbol{F} = m\boldsymbol{g} + 2m\boldsymbol{v} \times \boldsymbol{\omega} \quad \rightarrow \quad \boldsymbol{F} = m\boldsymbol{g} - 2m\boldsymbol{\omega} \times \boldsymbol{v}$$

Where the Coriolis force is defined as:

$$\boldsymbol{F}_{\boldsymbol{Coriolis}} = -2m\boldsymbol{\omega} \times \boldsymbol{v}$$

Now let,

$$\boldsymbol{g} = \boldsymbol{g}_{\boldsymbol{eff}} = (a_f - a_c)\widehat{\boldsymbol{g}}$$

So,

$$\boldsymbol{F} = m\boldsymbol{a}_{\boldsymbol{f}} - m\boldsymbol{\omega} \times \boldsymbol{\omega} \times \boldsymbol{r} - 2m\boldsymbol{\omega} \times \boldsymbol{v}$$

Poynting Theorem

$$\frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \boldsymbol{S} + \boldsymbol{g} \cdot \boldsymbol{J} = \boldsymbol{0}$$

$$P_{field} = \frac{\partial \boldsymbol{W}}{\partial t} + \nabla \cdot \boldsymbol{S} = -\boldsymbol{g} \cdot \boldsymbol{J}$$

Poynting Vector for EM	Poynting Vector for Gravitation
$\boldsymbol{S} = \frac{c_e}{4\pi} \boldsymbol{E} \times \boldsymbol{B}$	$\boldsymbol{S} = -\frac{c_g}{4\pi G} \boldsymbol{g} \times \boldsymbol{h}$

Fields for Uniform Motion

… for Electromagnetism

$$\boldsymbol{E}_v(\boldsymbol{R_P}, t) = \frac{e(1-\beta^2)\widehat{\boldsymbol{R}}_P}{R_P^2(1-\beta^2\sin^2\theta_P)^{3/2}}$$

$$\boldsymbol{B}_v(\boldsymbol{R_P}, t) = \boldsymbol{\beta} \times \boldsymbol{E}_v$$

… for Gravitation

$$\boldsymbol{g}_v(\boldsymbol{R_P}, t) = -\frac{Gm(1-\beta^2)\widehat{\boldsymbol{R}}_P}{R_P^2(1-\beta^2\sin^2\theta_P)^{3/2}}$$

$$\boldsymbol{h}_v(\boldsymbol{R_P}, t) = \boldsymbol{\beta} \times \boldsymbol{g}_v$$

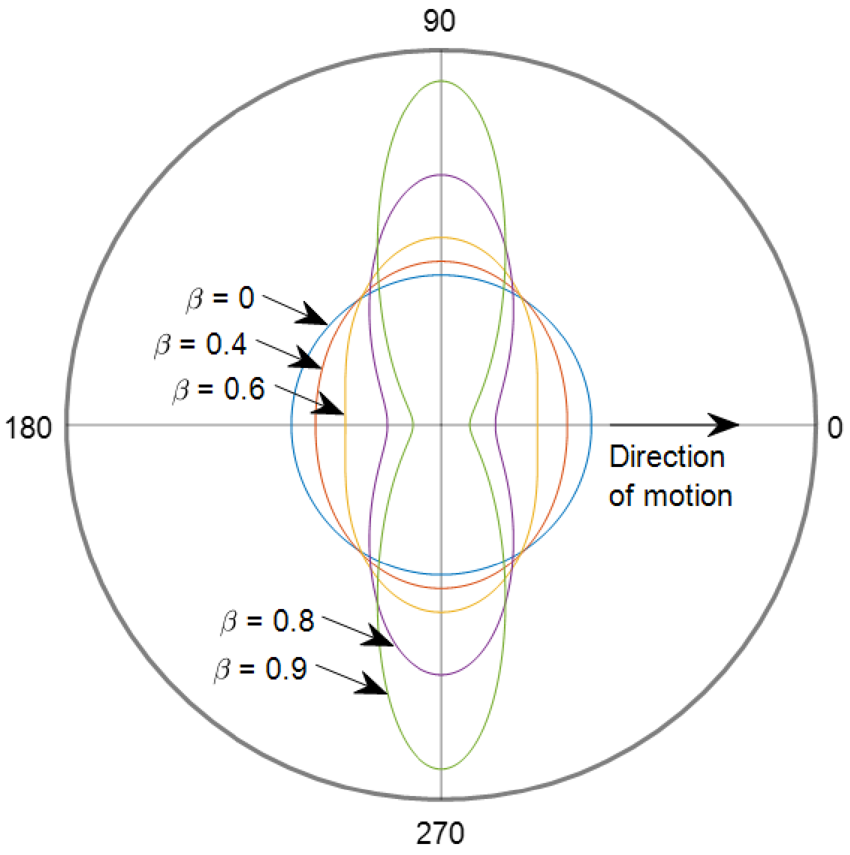


Figure 5

Fields for Uniform Motion (continued)

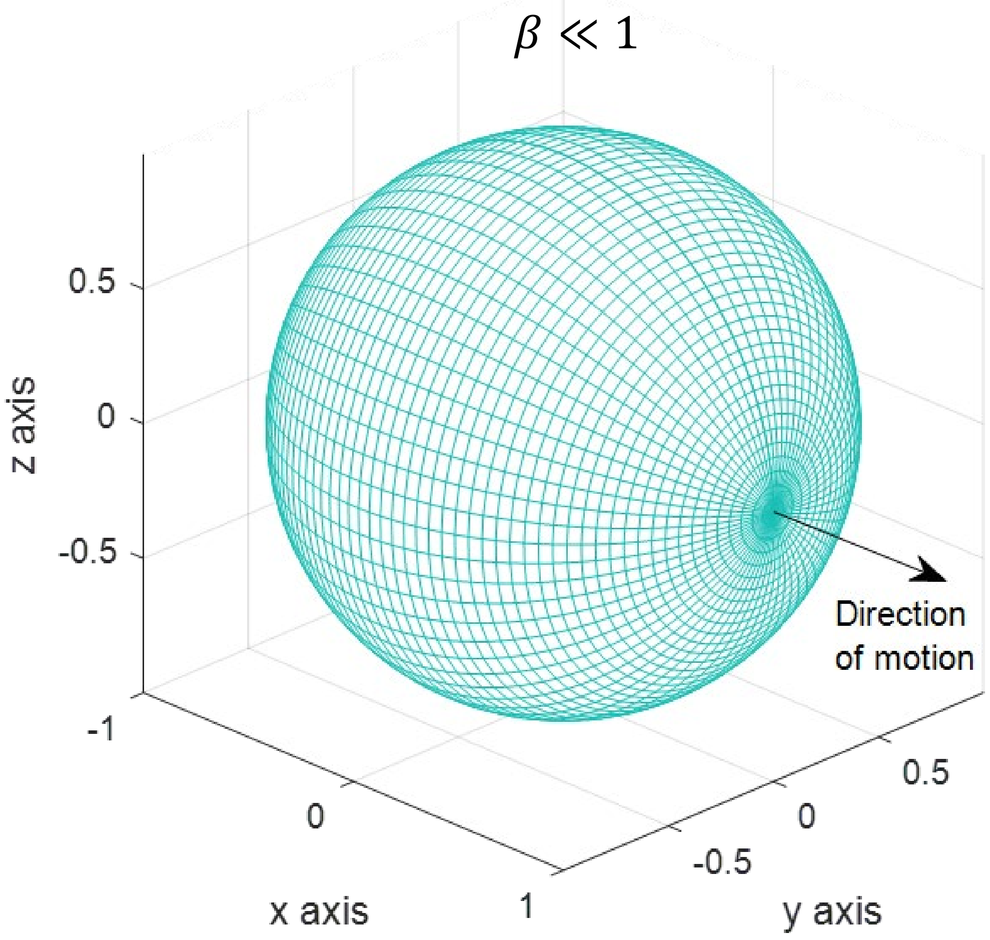


Figure 6

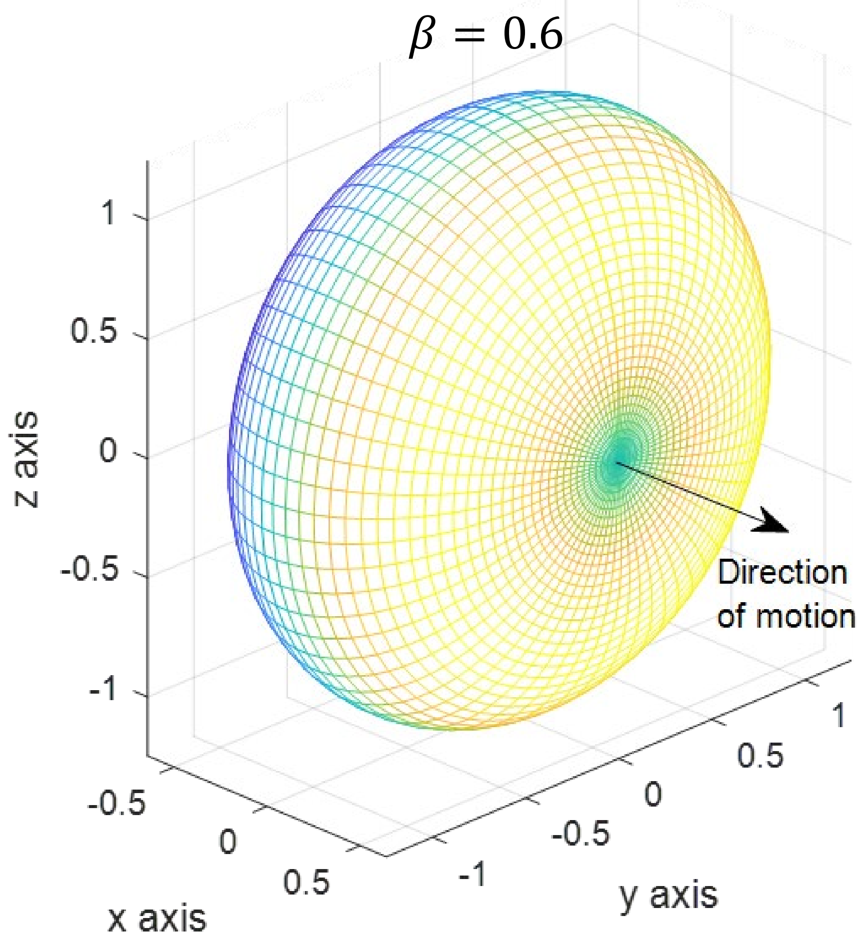


Figure 7

Radiation from Acceleration at Low Velocities

… for Electromagnetism

$$\frac{dP}{d\Omega} = (\boldsymbol{S_a} \cdot \widehat{\boldsymbol{R}})R^2 = \frac{e^2a^2}{4\pi c_e^3}\sin^2\theta$$

$$P = \frac{2e^2a^2}{3c_e^3}$$

… for Gravitation

$$\frac{dP}{d\Omega} = (\boldsymbol{S_a} \cdot \widehat{\boldsymbol{R}})R^2 = \frac{-Gm^2a^2}{4\pi c_g^3}\sin^2\theta$$

$$P = -\frac{2Gm^2a^2}{3c_g^3}$$

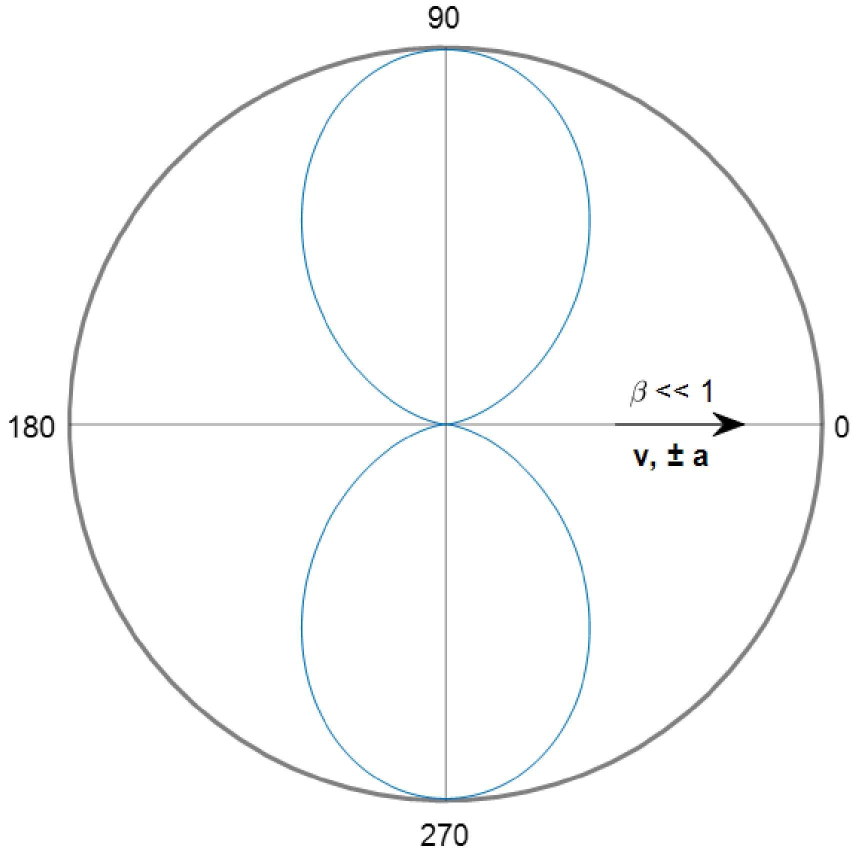


Figure 8

Radiation from Acceleration at Low Velocities

… for Electromagnetism

$$\frac{dP}{d\Omega} = (\boldsymbol{S_a} \cdot \widehat{\boldsymbol{R}})R^2 = \frac{e^2 a^2}{4\pi c_e^3} \sin^2\theta$$

$$P = \frac{2e^2 a^2}{3c_e^3}$$

… for Gravitation

$$\frac{dP}{d\Omega} = (\boldsymbol{S_a} \cdot \widehat{\boldsymbol{R}})R^2 = \frac{-Gm^2 a^2}{4\pi c_g^3} \sin^2\theta$$

$$P = -\frac{2Gm^2 a^2}{3c_g^3}$$

$\beta \ll 1$

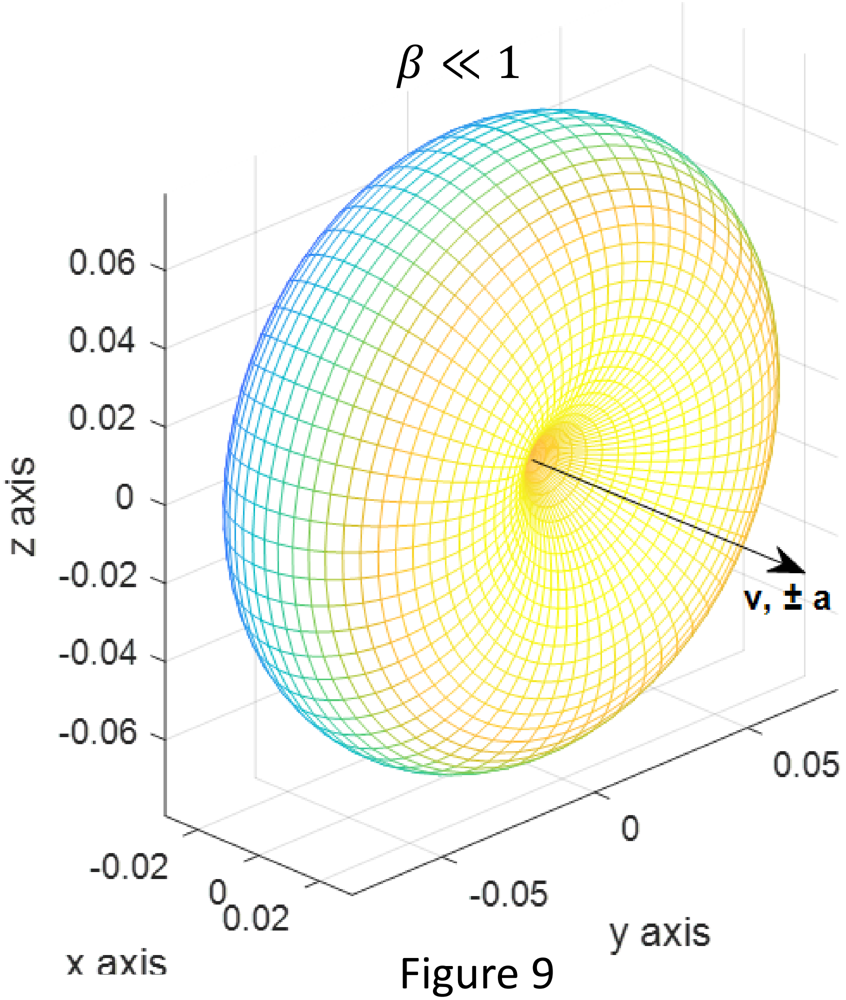


Figure 9

Gravitational Radiation from a Mass With Collinear Velocity and Acceleration

$$\frac{dP}{d\Omega} = \frac{-Gm^2a^2\sin^2\theta}{4\pi c_g^3(1-\beta\cos\theta)^5}$$

$$P = -\frac{2Gm^2a^2}{3c_g^3(1-\beta^2)^3}$$

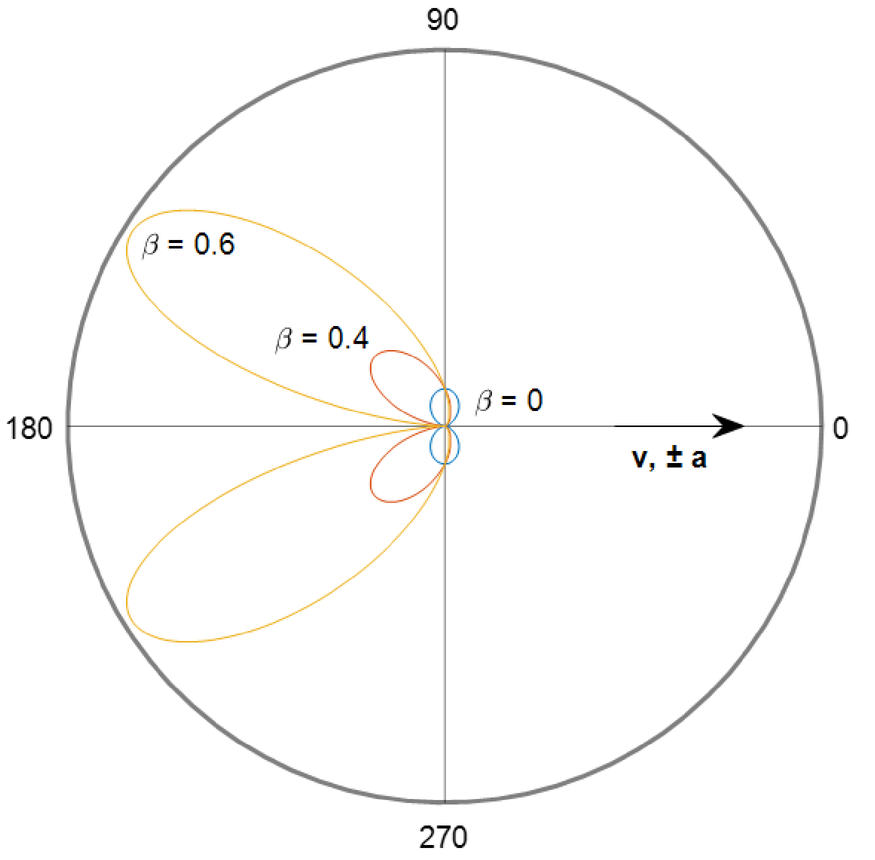


Figure 10

Gravitational Radiation from a Mass With Collinear Velocity and Acceleration (cont.)

$\beta = 0.6$

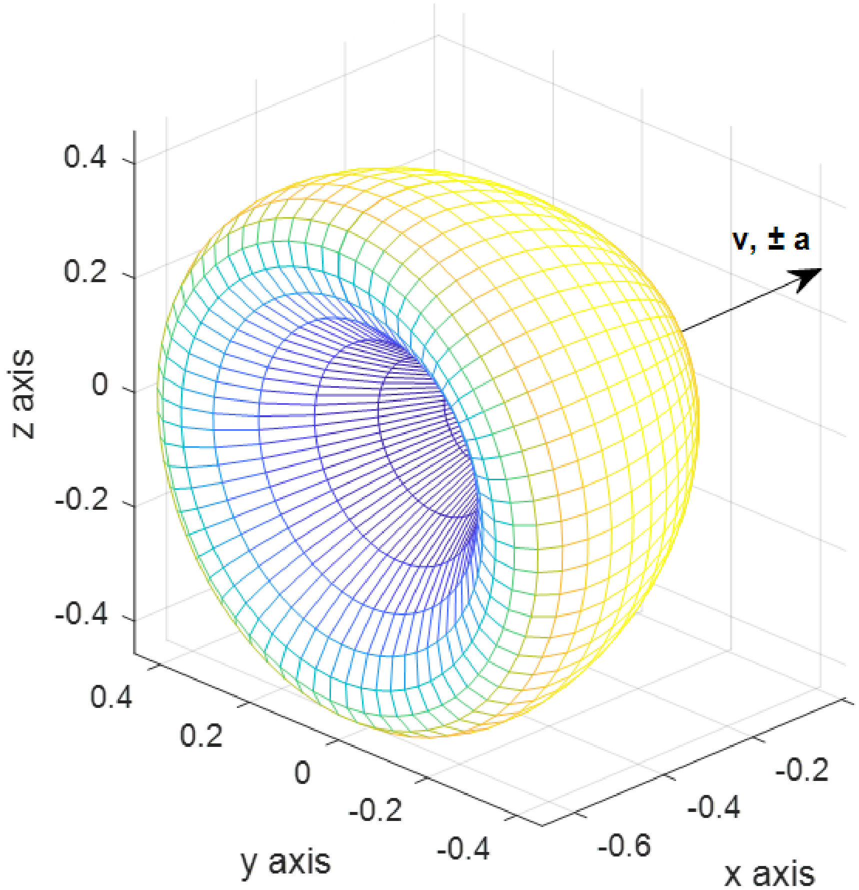


Figure 11

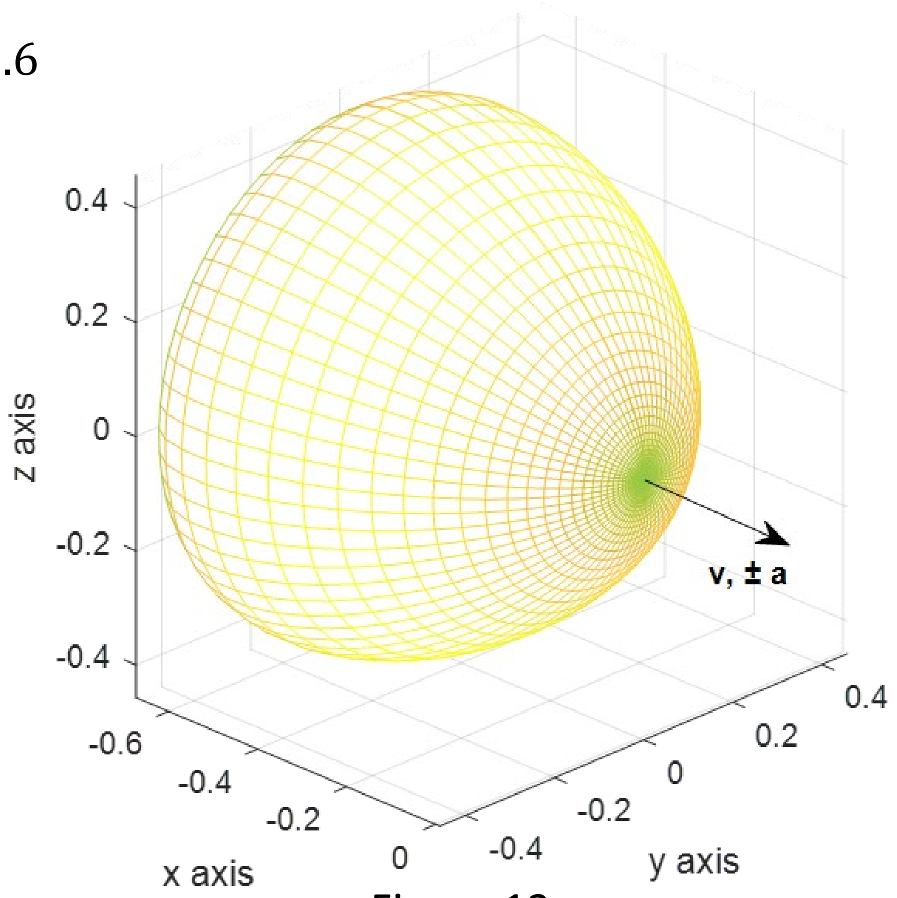


Figure 12

Radiation by Collinear Velocity & Acceleration

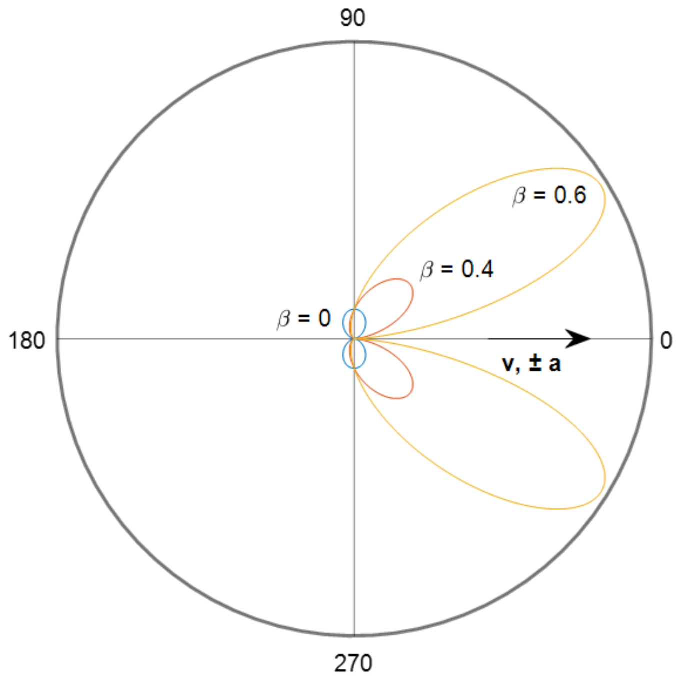


Figure 13

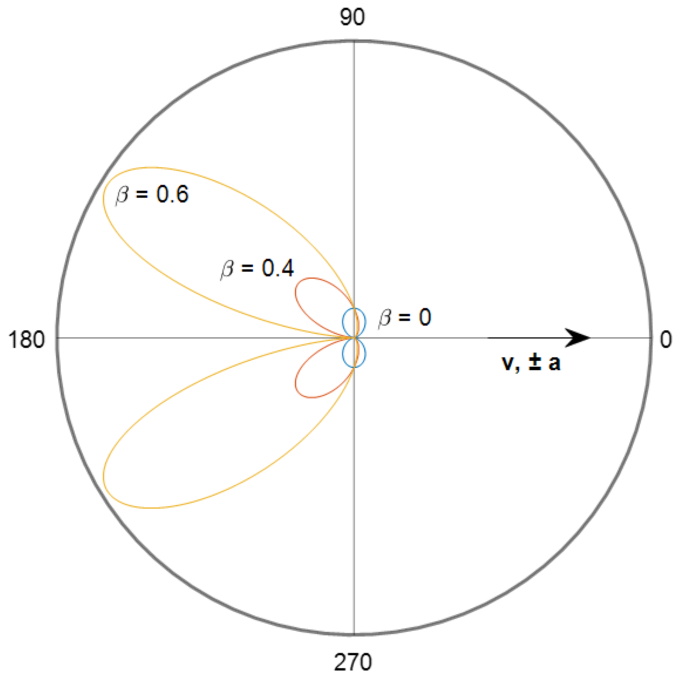


Figure 14

… for Electromagnetism	… for Gravitation
$\frac{dP}{d\Omega} = \frac{e^2 a^2 \sin^2\theta}{4\pi c_e^3 (1-\beta\cos\theta)^5} \rightarrow P = \frac{2e^2a^2}{3c_e^3(1-\beta^2)^3}$	$\frac{dP}{d\Omega} = \frac{-Gm^2 a^2 \sin^2\theta}{4\pi c_g^3 (1-\beta\cos\theta)^5} \rightarrow P = -\frac{2Gm^2a^2}{3c_g^3(1-\beta^2)^3}$

Gravitational Radiation Produced by a Mass in a Circular Orbit

$$\frac{dP}{d\Omega} = \frac{-Gm^2a^2[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi]}{4\pi c_g^3(1-\beta\cos\theta)^5}$$

$$P = -\frac{2Gm^2a^2}{3c_g^3(1-\beta^2)^2}$$

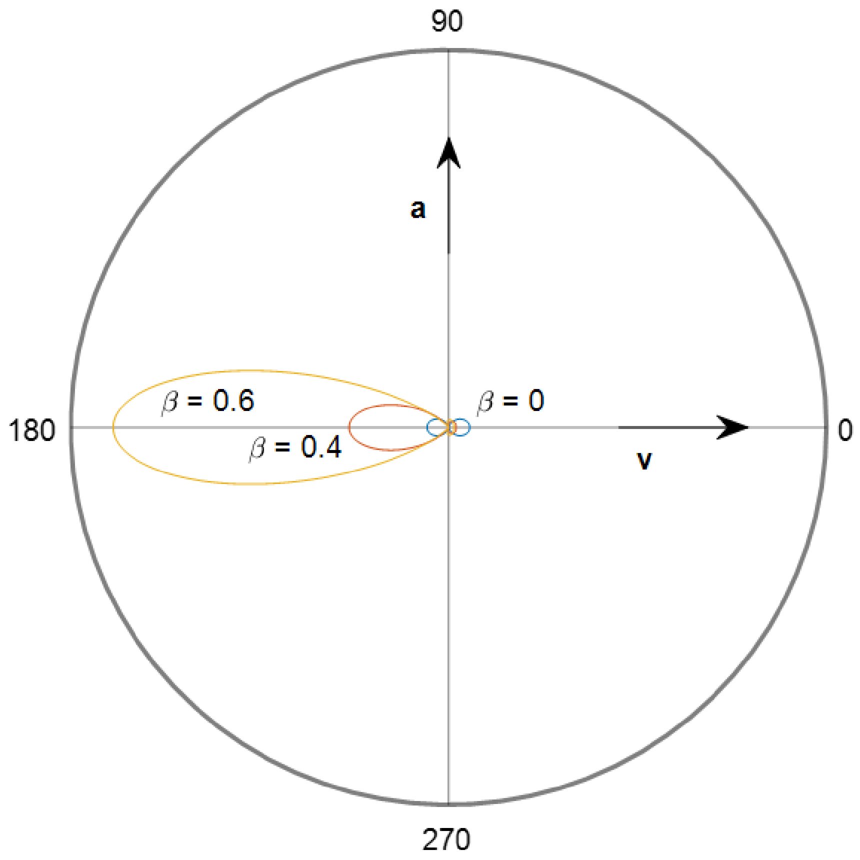


Figure 15

Gravitational Radiation Produced by a Mass in a Circular Orbit (continued)

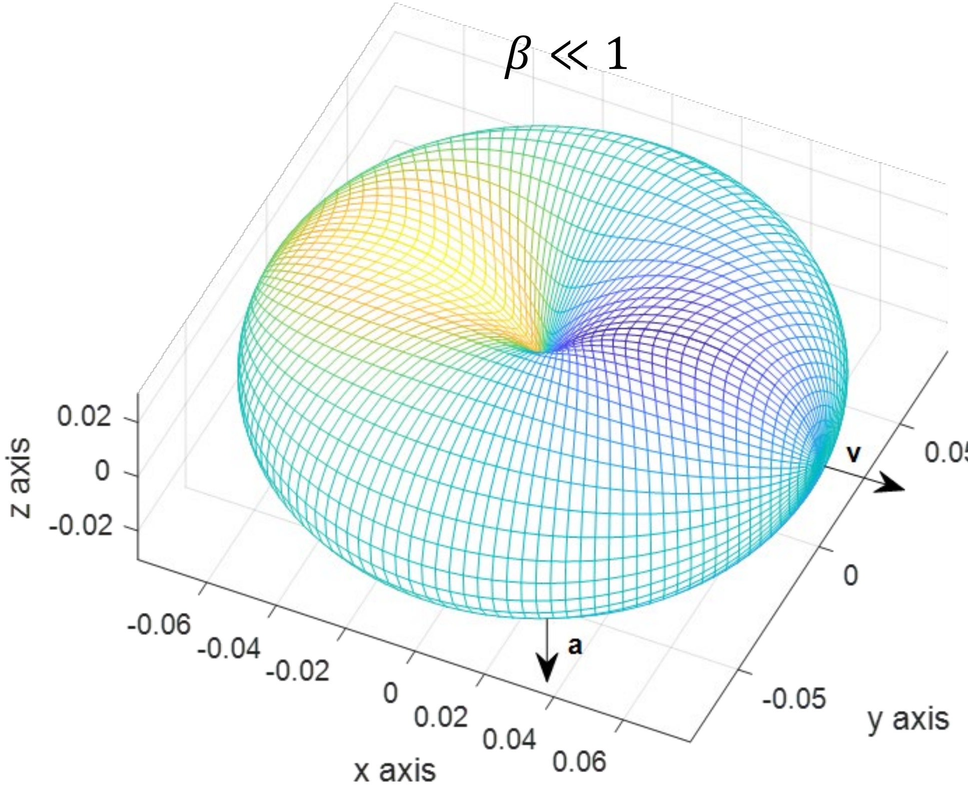


Figure 16

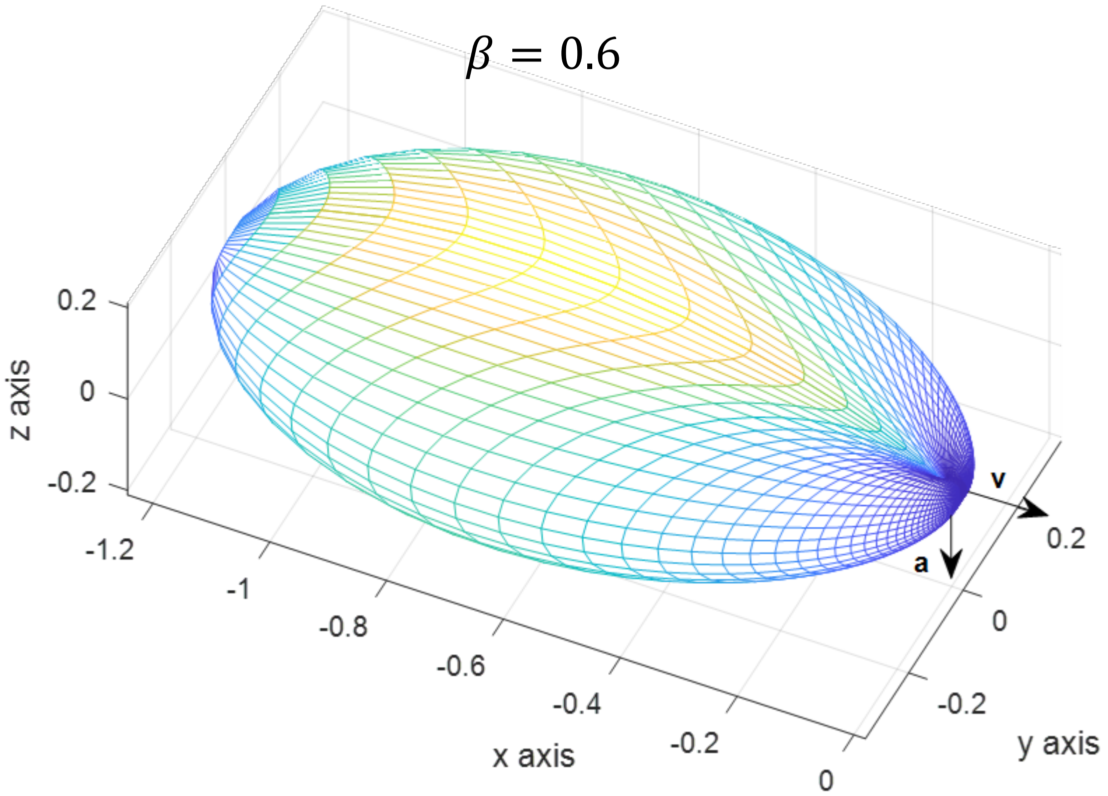


Figure 17

Radiation in a Circular Orbit

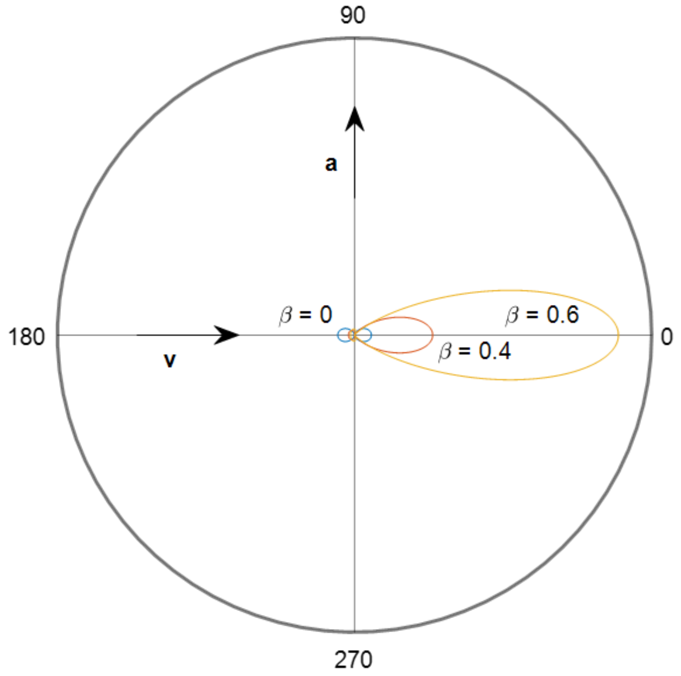


Figure 18

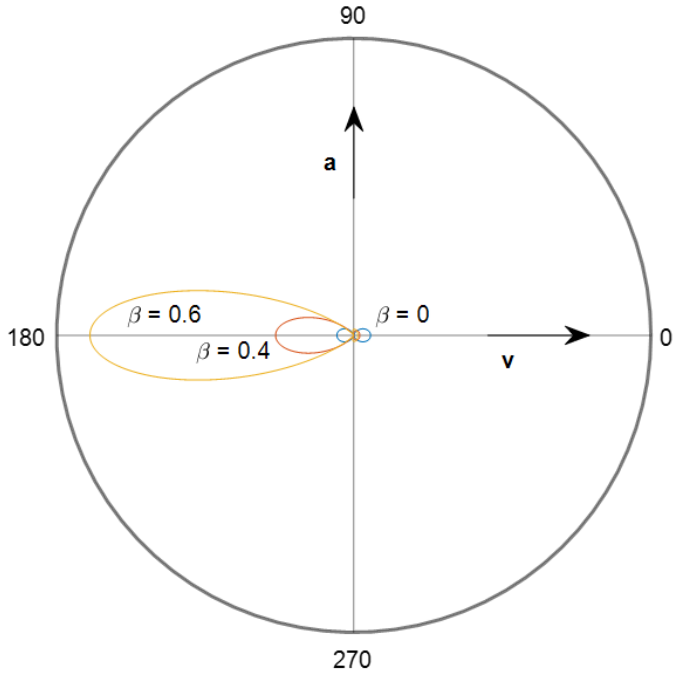


Figure 19

… for Electromagnetism

$$\frac{dP}{d\Omega} = \frac{e^2 a^2[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi]}{4\pi c_e^3(1-\beta\cos\theta)^5}$$

$$\rightarrow \; P = \frac{2e^2a^2}{3c_e^3(1-\beta^2)^2}$$

… for Gravitation

$$\frac{dP}{d\Omega} = \frac{-Gm^2 a^2[(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\varphi]}{4\pi c_g^3(1-\beta\cos\theta)^5}$$

$$\rightarrow \; P = -\frac{2Gm^2a^2}{3c_g^3(1-\beta^2)^2}$$

Gravitational Radiation Animation

- https://gravityvisualizer.vercel.app/

Conclusions

- Sources of gravitational fields are mass and momentum
- Field energy is negative
- Gravitational fields and radiation behave relativistically
- Gravitational radiation is produced by accelerated masses
- Gravitational radiation patterns mirror dipole patterns for electromagnetic radiation

Future Work

- Immediate Future
  - Determine precession for planetary orbits and compare to observation
  - Lense-Thirring precession
  - Develop a radiation instructional tool
- Foreseeable Future
  - Quantum gravity

References

- Heald, M,. and Marion, J,. “Classical Electromagnetic Radiation,” *Dover*, 2012
- Heaviside, O., “Electromagnetic Theory,” *The Electrician*, 1893, 31, pp. 281-282 & 359
- Hills, B,. “Gravito-electromagnetism and Mass Induction,” 2020
- Jefimenko, O., “Causality, Electromagnetic Induction, and Gravitation,” *Electret Scientific, 2000*

Acknowledgements

- Thesis Committee
- Engineering & Physics Department
- Jackson College of Graduate Studies
- RCSA Research Grant
- Kegan Johnson & Maddy Clark

Questions?